

Dessin d'Enfant: Drawing on a Torus and Other Surfaces

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A *dessin d'enfant* (“child’s drawing”) is a graph in which each vertex is colored black or white, edges do not intersect, and the two ends of every edge have different colors. Below are few examples on a sphere:

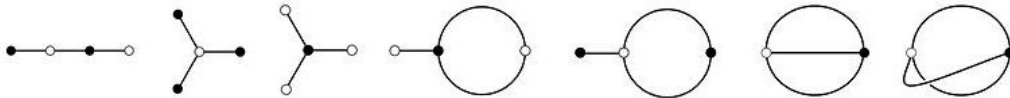


Figure 1: Dessin of degree 3 (nonintersecting on a sphere)

Dessins were first promoted in 1984 by Alexandre Grothendieck, one of the fathers of algebraic geometry, as a method of solving questions in arithmetic, algebraic geometry, complex analysis and other fields. These connections allow us to determine the existence of certain (covering) maps from a surface S to the Riemann-sphere, by determining the existence of a corresponding Dessin on S .

We will explore whether or not it is possible to draw certain dessin d'enfants on surfaces other than a sphere. For example, there are four families of regular dessin d'enfant on a torus, which correspond to regular tilings. Two are drawn in figure 2. However, it is not known whether it is possible to draw a dessin on a torus which divides it into hexagons with vertices of valency 3 except two black vertices, one with valency 2 and the other with valency 4.

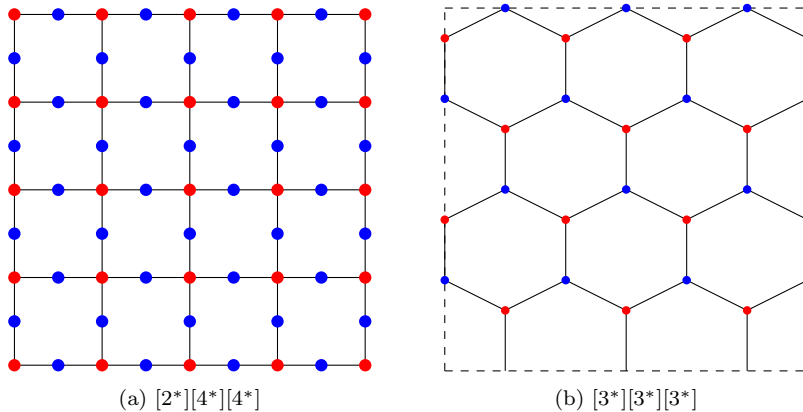


Figure 2: Two regular dessin on a torus

Prerequisites: no knowledge of algebraic geometry or complex analysis is needed. Some understanding of group theory and topology will be useful, but is not necessary. The project will be run in both Hebrew and English. If you like geometry, algebra, topology, combinatorics and discrete math, this is a good project for you.