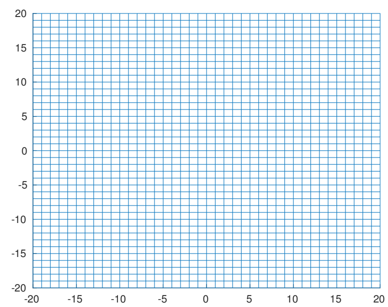
# Percolation and random graphs

Percolation theory is a model in mathematics and statistical physics which describes the behavior of connected components in random graph, and is the simplest model in which the phenomena of phase transition can be observed.

For example, consider the 2-dimensional Euclidean lattice $\mathbb{Z}^2$ with edges between any pair of points $(x_1, y_1), (x_2, y_2)$ such that $|x_1 - x_2| + |y_1 - y_2| = 1$. Here is an illustration of part of the Euclidean lattice.




For a parameter $p \in [0, 1]$, delete each of the edges independently with probability $1 - p$, and keep it with probability $p$. Here are a few examples:

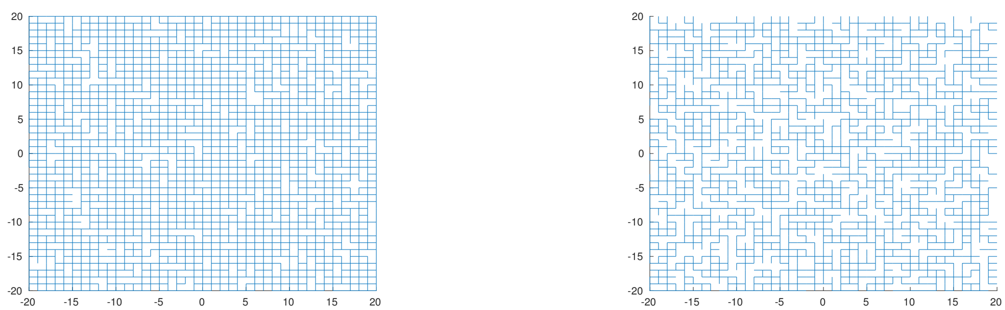


Figure 0.1: Percolation with $p = 0.9$ (on the left) and $p = 0.7$ (on the right)

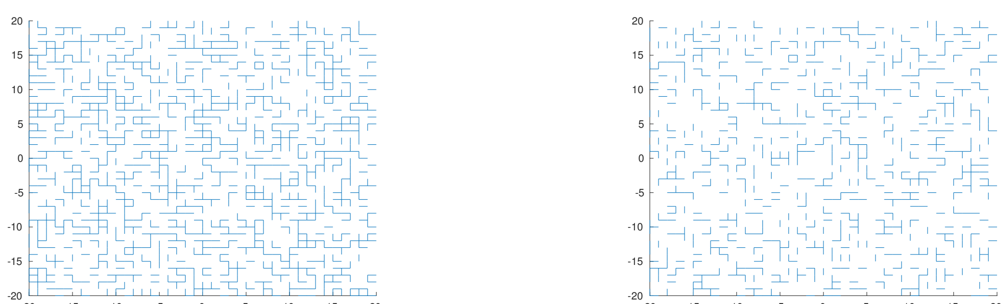


Figure 0.2: Percolation with $p = 0.4$ (on the left) and $p = 0.25$ (on the right)

Observing the pictures, one can see that the graph structure in the first two examples ($p$ sufficiently large) is very different than the graph structure in the last two examples ($p$ sufficiently small). For example, one may check that in the first two cases there is an open path from left to right, while in the other two there is no such path. In fact, there is a special value $p_c$, called the critical value, in which the behavior changes drastically. The existence of different behavior for different values of $p$ is called (the existence of) phase transition.

In the first part of the project, you will learn the precise definition of the notion of phase transition in percolation and find examples of graphs in which it occurs. Some time may be devoted to study and draw such graphs on a computer, as it provides useful insights. The second part will concern with understanding in more detail the structure of connected component in percolation.

Prerequisites: Basic course in probability. Some programming skills can be an advantage (but are not necessary).