UNIFORM DISTRIBUTION

INSTRUCTOR : AMOS NEVO

If asked to produce a sequence of finite sets of points A_n which becomes uniformly distributed in the unit interval, you might suggest dividing the unit interval to n equal sub-intervals and taking A_n to be the set of division points.

If asked to produce to produces a sequence of finite sets B_n of points which becomes uniformly distributed in the unit square, you might suggest dividing the unit square to n^2 sub-squares of side-length $\frac{1}{n}$ and taking B_n to be the set of their vertices.

But what would you do if asked to produces a sequence of finite sets of points which becomes uniformly distributed in the unit sphere ?

This question is a gateway to several important current subjects in contemporary mathematics and there is a lot of active research devoted to many aspects of it. Interestingly, although the problem is an analytical one, it has deep and surprising connections to several aspects of group theory and algebra.

In our project we will begin by discussing the notion of uniform distribution in various spaces, and some of the wide range of tools that are utilized in its study. We will then describe what is known about uniform distribution on the unit sphere and present some problems related to it.

Prerequisites. We will assume familiarity with the concepts and results of the basic undergraduate courses in infinitesimal calculus and linear algebra, including familiarity with basic group theory. Familiarity with basic undergraduate functional analysis including an introduction to Fourier series will be an advantage, but is not a pre-requisite.