

PDE Day with Louis Nirenberg

ABSTRACTS

Henri Berestycki, EHESS, PSL - Research University Paris

The effect of domain shape on reaction-diffusion equations

I will discuss some reaction-diffusion equations of bistable type motivated by biology and medicine. The aim is to understand the effect of the shape of the domain on propagation or on blocking of advancing waves. I will first describe the motivations of these questions and present a result about the existence of generalized “transition waves”. I will then discuss various geometric conditions that lead to either blocking, or partial propagation, or complete propagation. These questions involve new qualitative results for some non-linear elliptic and parabolic partial differential equations. Much of the qualitative information is gleaned thanks to the moving plane and sliding methods in the spirit of earlier work with Louis Nirenberg. I report here on joint work with Juliette Bouhours and Guillemette Chapuisat.

Isabeau Birindelli, Sapienza Università di Roma

Maximum principle, eigenvalues and qualitative properties for fully nonlinear elliptic equations

We shall present different results that link the maximum principle, the sign of the “principal eigenvalue” and the symmetry of solutions for operators that are elliptic, degenerate elliptic and fully nonlinear.

Hàim Brezis, Rutgers University

Nonlocal approximations of the BV norm

I will present new results concerning the approximation of the BV- norm by nonlocal, nonconvex functionals. The mode of convergence is extremely delicate and numerous problems remain open. De Giorgi’s concept of Gamma convergence illuminates the situation but also introduces mysterious novelties. The original motivation comes from Image Processing. This is joint work with Hoai-Minh Nguyen.

Yanyan Li, Rutgers University

Homogeneous solutions of stationary Navier-Stokes equations with isolated singularities on the unit sphere

We classify all (-1) -homogeneous axisymmetric no swirl solutions of incompressible stationary Navier-Stokes equations in three dimension which are smooth on the unit sphere minus the south pole, parameterize them as a two dimensional surface with boundary, and analyze their pressure profiles near the north pole. Then we prove that there is a curve of (-1) -homogeneous axisymmetric solutions with nonzero swirl, having the same smoothness property, emanating from every point of the interior and one part of the boundary of the solution surface. Moreover, we prove that there is no such curve of solutions for any point on the other part of the boundary. We also establish asymptotic expansions for every (-1) -homogeneous axisymmetric solutions in a neighborhood of the singular point on the unit sphere. This is joint work with Li Li (Harbin Institute of Technology) and Xukai Yan (Rutgers University).

Moshe Marcus, Technion

Schrödinger equations and applications to semilinear problems

We consider Schrödinger equations of the form $-\Delta u + Vu = 0$ in a bounded Lipschitz domain $D \subset \mathbb{R}^N$. We assume that the potential $V \in C(D)$ is positive and satisfies the condition $V(x) \leq a \text{dist}(x, \partial D)^{-2}$, $\forall x \in D$, where a is a constant. We discuss properties of the linear equation - including estimates of Green functions - under various additional conditions on V . In continuation we discuss applications to the study of semilinear problems of the form $-\Delta u + Vu + g(u) = 0$ where $g \in C(\mathbb{R})$ is monotone increasing and superlinear on \mathbb{R}_+ .