Problem 1. Let $X = L^{m,p}(\mathbb{R}^n)$. For $E \subset \mathbb{R}^n$, let $X(E)$ be the space of restrictions to $E$ of functions in $X$.

If $p > n$, it is known that there exists a linear extension operator $T : X(E) \to X$, with norm at most $C(m,n,p)$.

Also, if $p > n$ and $E$ is finite ($\#(E) = N$), then $\|f\|_{X(E)}^p$ is known to be comparable to a sum of $p^{th}$ powers of $O(N)$ linear functionals $\xi_1, \ldots, \xi_L$ acting on $f$. The operator $T$ and the functionals $\xi_1, \ldots, \xi_L$ can be taken to have a sparse structure, and there are efficient algorithms to compare them. See papers by Fefferman-Israel-Luli on the arXiv.

What if $\frac{m}{m} < p \leq n$? We know almost nothing about this case.

Problem 2. Let $X$ be a metric space and let $Y$ be a Banach space.

For each $x \in X$, let $K(x)$ be a convex subset of $Y$ with dimension at most $D$.

Suppose that for each $S \subset X$ with at most $2^{D+1}$ points, there exists a map $F^S : S \to Y$ with Lipschitz constant at most 1, such that

$$F^S(x) \in K(x) \quad \text{for} \quad x \in S.$$ 

Prove that there exists $F : X \to Y$ with Lipschitz constant at most $C(D)$ such that

$$F(x) \in K(x) \quad \text{for all} \quad x \in X.$$ 

This question is due to Pavel Shvartsman, motivated by his joint works with Yuri Brudnyi on Whitney’s extension problem.

Problem 3. Let $X$ be a finite metric space.

Give an algorithm to decide whether $X$ can be imbedded in a $D$-dimensional connected Riemannian manifold with sectional curvatures and diameter bounded above by $O(1)$, and with injectivity radius bounded below by 1.

This question comes from Slava Kurylov, Matti Lassas, Sergei Ivanov and Hari Narayanan.

Problem 4. Given a function $f : E \to \mathbb{R}$ decide whether $f$ extends to a $C^\infty$ function.
**Problem 5.** Fix $m \geq 1$, and let $0 \in E \subset \mathbb{R}^n$.

Let $I(E)$ be the ideal of all $m$-jets at 0 of functions (in $C^m$, or maybe in $C^\infty$) vanishing in $E$. Then $I(E)$ is an ideal in the ring of $m$-jets at 0.

- Which ideals arise as $I(E)$ for some $E$?
- Does every $I(E)$ arise already as $I(\hat{E})$ for a semialgebraic set $\hat{E}$?

These questions are due to Nahum Zobin.

**Problem 6.** Let $E$ be compact.

For each $x \in E$, let $H(x) \subset \text{Ring of } m\text{-jets at } x$ be a coset of an ideal.

Suppose that

- There exists $F \in C^m(\mathbb{R}^n)$ such that
  
  $J_x(F) \equiv m\text{-jet of } F \text{ at } x \text{ lies in } H(x) \text{ for all } x \in E$;

and

- $\{(x,P) : x \in E, P \in H(x)\}$ is semialgebraic.

Does there exist $F \in C^m(\mathbb{R}^n)$ \underline{semialgebraic} such that

$J_x(F) \text{ lies in } H(x) \text{ for all } x \in E$?

(I’m not sure who first asked this question.)