SMOOTHLY ORIENTING CUBES

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Motivation

Tiling the volume bounded by a closed surface with cuboid elements, or *hexahedral meshing* (Fig. 1), is an important problem in numerical computing, with multiple applications in numerical simulation and computer graphics. One approach for tackling this problem is to first define a sample of cube *orientations* at different points in the volume, and then *smoothly interpolate* these orientations to the remaining locations. Thus, the following two questions arise: (1) how to define a meaningful notion of *distance* between cube orientations, and (2) how to smoothly interpolate between such orientations.

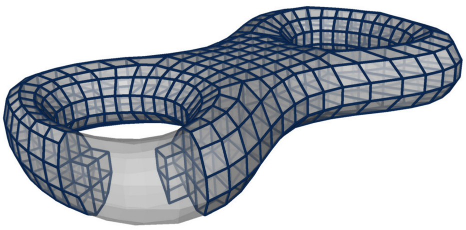

Figure 1. Tiling the volume bounded by a closed surface with deformed cubes. From [NRP11]

Problem Description

Distance. Given two orientations $R_1, R_2 \in \mathrm{SO}(3)$ define a distance $d(R_1, R_2) \in \mathbb{R}$ such that it is *invariant to the orientation preserving symmetries of the cube.* Specifically, let O be the octahedral group, then $d(R_1, R_2) = d(R_1 \circ o_1, R_2 \circ o_2)$ for any $o_1, o_2 \in O$.

Interpolation. Given a uniform grid on the unit square and an assignment of cube orientations R_k to the vertices on the boundary, find an assignment of orientations in the interior, such that the sum of the squared distances $\sum_{(i,j)\in E} d^2(R_i, R_j)$ on the edges E of the grid is minimized.

Project Goals

The goals of the project are to investigate the two aforementioned problems in two and three dimensions, experiment with various existing definitions and techniques, and get new insights into possible solutions. We will begin by exploring the two dimensional case, brainstorm about different solutions, and then learn about the trade-offs in existing solutions for this setting [KNP07, BZK09]. Then we will look into the 3D case [SVB17], play with toy examples, and try to come up with new ideas for representations and distances.

Prerequisites. The project will involve experimental math and Matlab coding, and will most likely not involve proving theorems. Background in group theory could come in handy and experience with Matlab is a must.

References

[BZK09] David Bommes, Henrik Zimmer, and Leif Kobbelt. Mixed-integer quadrangulation. *ACM Transactions On Graphics (TOG)*, 28(3):77, 2009.

[KNP07] Felix Kälberer, Matthias Nieser, and Konrad Polthier. Quadcover-surface parameterization using branched coverings. In *Computer Graphics Forum*, volume 26, pages 375–384. Wiley Online Library, 2007.

[NRP11] Matthias Nieser, Ulrich Reitebuch, and Konrad Polthier. Cubecover–parameterization of 3d volumes. In *Computer Graphics Forum*, volume 30, pages 1397–1406. Wiley Online Library, 2011.

[SVB17] Justin Solomon, Amir Vaxman, and David Bommes. Boundary element octahedral fields in volumes. In *Transactions on Graphics*, 2017.