Book of Abstracts

October 16

Local attractors of Newton-type methods for constrained equations with nonisolated solutions

Andreas Fischer

For a smooth system of equations subject to convex constraints with nonempty interior, we investigate the behavior of a family of Newton-type methods in the neighborhood of a nonisolated solution. There are several results on the superlinear convergence of such methods if a Lipschitzian error bound at the solution holds. In contrast to this, we are interested in situations where this error bound is violated. Instead of a Lipschitzian error bound, we assume that the equation mapping is 2-regular at some specfic solution with respect to a direction in the null space of the Jacobian. If this direction is strictly feasible, we can show that there is a domain of starting points from which the Newton-type methods are well-defined and converge linearly to the specific solution, interestingly despite the fact that this solution is nonisolated. Finally, the application of the result to reformulations of the nonlinear complementarity problem will be sketched.

The talk is based on joint work with Alexey F. Izmailov (Moscow State University) and Mikhail V. Solodov (IMPA, Rio de Janeiro).

Avoiding critical multipliers and slow convergence of primal-dual algorithms

Boris S. Mordukhovich

In this talk we introduce the notions of critical and noncritical multipliers for subdifferential variational systems extending to a general framework the corresponding notions by Izmailov and Solodov developed for classical Karush-Kuhn-Tucker (KKT) systems. It has been well recognized that critical multipliers are largely responsible for slow convergence of major primal-dual algorithms of optimization. The approach of this paper allows us to cover KKT systems arising in various classes of smooth and nonsmooth problems of constrained optimization including composite optimization, minimax problems, etc. Concentrating on a polyhedral subdifferential case and employing recent results of second-order subdifferential theory, we obtain complete characterizations of critical and noncritical multipliers via the problem data. It is shown that noncriticality is equivalent to a certain error bound for a perturbed variational system and that critical multipliers can be ruled out by full stability of local minimizers in problems of composite optimization. For the latter class we establish the equivalence between noncriticality of multipliers and robust isolated calmness of the associated solution map and then derive explicit characterizations of these notions via appropriate second-order sufficient conditions. It is finally proved that the Lipschitz-like property of solution maps yields their robust isolated calmness.

The talk is based on joint work with Ebrahim Sarabi (Miami University, Oxford, OH).

Complexity bounds for primal-dual methods minimizing the model of objective function

Yurii Nesterov

We provide Frank-Wolfe (Conditional Gradients) method with a convergence analysis allowing to approach a primal-dual solution of convex optimization problem with composite objective function. Additional properties of complementary part of the objective (strong convexity) significantly accelerate the scheme. We also justify a new variant of this method, which can be seen as a trust-region scheme applying the linear model of objective function. Our analysis works also for a quadratic model, allowing to justify the global rate of convergence for a new second-order method. To the best of our knowledge, this is the first trust-region scheme supported by the worst-case complexity bound

Individual demand, utility maximization, and systems of differential equations

Yakar Kannai

Individual consumers demand goods and services so as to maximize their satisfaction (represented by utility functions) subject to their budget constraint. Consider the multi-period cases where (i) the decision maker ignores future prices ("myopia") or (ii) tastes change in time. Both cases lead to (over-determined systems) of differential equations for the utility functions.

Transversality properties of pairs of sets and alternating projections

Alexander Kruger

Several kinds of 'regular' arrangement of a pair of sets near a point in their intersection will be discussed: *transversality, subtransversality* and *intrinsic transversality*. Such regular intersection properties are crucial for the validity of qualification conditions in optimization as well as subdifferential, normal cone and coderivative calculus, and convergence analysis of computational algorithms. Dual characterizations of the properties in terms of *Fréchet*, *limiting* and *proximal normals* will be provided.

References:

D. Drusvyatskiy, A. D. Ioffe and A. S. Lewis, *Transversality and alternating projections for nonconvex sets*, Found Comput Math **15** (2015),1637–1651

A. Y. Kruger and N. H. Thao, *Regularity of collections of sets and convergence of inexact alternating projections*, J. Convex Anal. **23** (2016), no. 3, 823–847.

A. Y. Kruger, D. R. Luke and N. H. Thao, *Set regularities and feasibility problems*, Math. Program., DOI 10.1007/s10107-016-1039-x.

A. Y. Kruger, D. R. Luke and N. H. Thao, *About subtransversality of collections of sets*, Set-Valued Var. Anal., DOI 0.1007/s11228-017-0436-5.

A. Y. Kruger, About intrinsic transversality of pairs of sets, arXiv:1701.08246.

On a convexity principle with applications to nonconvex optimization

Amos Uderzo

In [2] it was proved that any C1;1 smooth mapping between Hilbert spaces, which is metrically regular around a reference point, carries balls centered at that point to convex sets, provided that their radius is small enough. Such a notable geometric property comes as a result of a synergic interplay between the rotundity property of the underlying space, the openness behavior near a point where a mapping is metrically regular, and the asymptotic behavior of the linear approximation of a sufficiently smooth mapping with Lipschitz derivative. The above convexity principle provides a local tool to face the long-standing problem of characterizing the images of convex sets under nonlinear mappings. It found several promising applications in linear algebra, optimal control theory and optimization (see [1, 2, 3, 4]). In the present talk, some recent extensions and limits of the Polyak convexity principle are discussed. Applications to nonconvex constrained optimization are considered. In particular, relevant consequences on such issues as global solution existence, Lagrangian duality and value-function analysis are presented.

References

1. B.T. Polyak, Convexity of quadratic transformations and its use in control and optimization, J. Optim. Theory Appl. 99 (1998), no. 3, 553-583.

2. B.T. Polyak, Convexity of Nonlinear Image of a Small Ball with Applications to Optimization, Set-Valued Anal. 9 (2001), 159-168.

3. B.T. Polyak, Local programming, Zh. Vychisl. Mat. Mat. Fiz. 41 (2001), no. 9, 1259-1266.

4. B.T. Polyak, The convexity principle and its applications, Bull. Braz. Math. Soc. (N.S.) 34 (2003), no. 1, 59-75.

An optimal control example arising in the singular perturbations limit

Zvi Artstein

We examine a specific optimal control problem that arises as the singular limit of an optimal control problem with a small parameter. The process that yields such examples will be displayed.

Saddle-point equilibrium sequence in singular infinite horizon zero-sum linear-quadratic differential game with state delays

Valery Y. Glizer

We consider an infinite horizon zero-sum linear-quadratic differential game. The differential equation of dynamics in this game has delays (point-wise and distributed) in the state variable. The case where the game's cost functional does not contain a control cost of the minimizing player (the minimizer) is treated. This feature of the cost functional means that the game under consideration is singular. For this game, definitions of the saddle-point equilibrium and game value are proposed. To obtain these saddle-point equilibrium and game value, a regularization method is applied. Namely, we associate the singular game with a new differential game for the same equation of dynamics. The cost functional in the new game is the sum of the original cost functional and an infinite horizon integral of the square of the minimizer's control with a small positive weight coefficient. This new game is regular, and it is a cheap control game. Using the conditions for the existence of a state-feedback saddle point in this cheap control game, its solution is reduced to solution of a hybrid set of Riccati-type matrix equations with deviating arguments and with indefinite quadratic terms. One of these equations is algebraic, while two others are first order ordinary and partial differential equations. This set is subject to given boundary conditions. Due to the presence of the small positive parameter in the cost functional of the cheap control game, the set of Riccati-type equations is perturbed by this small parameter. Subject to proper assumptions, an asymptotic expansion of a stabilizing solution to this set of equations is constructed and justified. Using this asymptotic expansion, the existence of the saddle-point equilibrium and the value of the original (singular) game is established, and their expressions are derived. Note, that a particular (undelayed) case of the original game was analyzed in the work [1]. In the work [2], an undelayed case of a singular infinite horizon linear-quadratic differential game was considered for a more general form of the cost functional. Under weaker conditions than in this talk and in [1], the upper value of the considered singular game was studied in [2].

References

[1] V. Y. Glizer, O. Kelis, Singular Infinite Horizon Zero-Sum Linear-Quadratic Differential Game: Saddle-Point Equilibrium Sequence. Numerical Algebra, Control and Optimization, Vol. 7, No. 1, pp. 1-20, 2017.

[2] V. Y. Glizer, O. Kelis, Upper Value of a Singular Infinite Horizon Zero-Sum Linear-Quadratic Differential Game. To appear in Pure and Applied Functional Analysis, Vol. 3, 2018.

The "hot start" phenomenon in convex optimization

Roman Polyak

The main idea of the interior point methods (IPMs) is moving from one "warm start" to another "warm start" mimicking the central path. We will discuss the advantages and limitations of the IPMs, then we recall the nonlinear rescaling (NR) scheme and show that it leads to the "hot start" phenomenon.

The fundamental difference between the "warm" and the "hot" starts will be discussed. In particular, in contrast to IPMs the NR scheme requires not more but less computational effort per extra digit of accuracy toward the end of the process. It allows solving difficult convex optimization (CO) problems with high accuracy.

We conclude by considering the primal-dual NR approach, which under standard second order sufficient optimality condition, leads to CO methods with asymptotic quadratic rate.

The area where the quadratic rate occurs will be characterized.

Speeding-up convergence via sequential subspace optimization (SESOP)

Michael Zibulevsky

We present the SESOP framework for smooth large-scale unconstrained, composite and stochastic optimization. It provides the worst-case optimality, combined with the average efficiency of Conjugate Gradient type methods. We explore SESOP combination with the parallel coordinate descent (PCD), the separable surrogate function method (SSF), multigrid and stochastic optimization methods, obtaining the state of the art results in the areas of sparse signal enhancement, compressive sensing, computed tomography, support vector machines, and deep learning.

October 17

Phase retrieval with application to optical imaging

Yonina Eldar

The problem of phase retrieval, namely the recovery of a function given the magnitude of its Fourier transform - arises in various fields of science and engineering, including electron microscopy, crystallography, astronomy, and optical imaging. Due to the loss of Fourier phase information, this problem is generally ill-posed. In this talk we review several modern methods for treating the phase retrieval problem including matrix lifting, structured illumination and short-time Fourier measurements. We also consider techniques that exploit sparsity on the input together with contemporary optimization tools to further facilitate recovery. We then illustrate the use of these methods in several different problems arising in optical imaging.

Feasible roundings for granular optimization

Oliver Stein

We introduce a new technique to generate good feasible points of mixed-integer nonlinear optimization problems which are granular in a certain sense. Finding a feasible point is known to be NP hard even for mixed-integer linear problems, so that many construction heuristics have been developed. We show, on the other hand, that efficiently solving certain purely continuous optimization problems and rounding their optimal points leads to feasible points of the original mixed-integer problem, as long as the original problem is granular. For the objective function values of the generated feasible points we present computable a-priori and a-posteriori bounds on the deviation from the optimal value, as well as efficiently computable certificates for the granularity of a given problem.

Computational examples for several problems from the MIPLIB libraries illustrate that our method is able to outperform standard software. A post processing step to our approach, using integer line search, further improves the results.

On some new methods to derive necessary and sufficient optimality conditions in vector optimization

Marius Durea, Radu Strugariu, Christiane Tammer

The aim of this talk is to address new approaches, in separate ways, to necessary and, respectively, sufficient optimality conditions in constrained vector optimization. In this respect, for the necessary optimality conditions that we derive, we use a kind of vectorial penalization technique, while for the sufficient optimality conditions we make use of an appropriate scalarization method. In both cases, the approaches couple a basic technique (of penalization or scalarization, respectively) with several results in variational analysis and optimization obtained by the authors in the last years. These combinations allow us to arrive to optimality conditions which are, in terms of assumptions made, new.

Finite element methods for optimal control problems with measures

Boris Vexler

In this talk we discuss optimal control problems subject to parabolic equations, where the support of the control is potentially of measure zero. This includes sparse optimal control problems [1], where the control variable lies in a measure space and problems with pointwise controls [4, 5]. Such formulations provide among other things an elegant way to attack problems of optimal source placement as well as point source identification problems. For this type of problems, we consider finite element discretizations in space and time and derive a priori error estimates. The main technical tools are recently established discrete maximal parabolic regularity [2] and pointwise best approximation results [3].

References

[1] K. Kunisch, K. Pieper, and B. Vexler. Measure valued directional sparsity for parabolic optimal control problems. SIAM J. Control Optim., 52(5):3078–3108, 2014.

[2] D. Leykekhman and B. Vexler. Discrete maximal parabolic regularity for Galerkin finite element methods. Num. Math., 135(3):923–952, 2017.

[3] D. Leykekhman and B. Vexler. Pointwise best approximation results for Galerkin finite element solutions of parabolic problems. SIAM J. Numer. Anal., 54(3):1365–1384, 2016.

[4] D. Leykekhman and B. Vexler. Optimal a priori error estimates of parabolic optimal control problems with pointwise control. SIAM J. Numer. Anal., 51(5):2797–2821, 2013.

[5] D. Leykekhman and B. Vexler. A priori error estimates for three-dimensional parabolic optimal control problems with pointwise control. SIAM J. Control Optim., 4(5):2403–2435, 2016.

Simple bilevel programming and extensions: theory and algorithms

Stephan Dempe, Nguyen Dinh, Joydeep Dutta, Tanushree Pandit

The simple bilevel optimization problem consists of minimizing a convex function f over the solution set of a second, convex optimization problem (the inner problem). This problem has been formulated e.g. in [3], it is closely related to bilevel optimization problems and has similar properties. To investigate this convex optimization problem, the inner problem needs to be transformed. This can be done using the Karush-Kuhn-Tucker conditions of the inner problem resulting in a so-called Mathematical Program with Equilibrium Constraints, using the optimal value of the inner problem or applying a variational inequality leading to a semi-infinite optimization problem. Applying variational analysis to the first problem, a necessary optimality condition is obtained which is not sufficient [1]. Results from semiinfinite optimization or the use of a gap function for the variational inequality can be applied to derive necessary and sufficient optimality conditions for the investigated problem. As a regularity condition we need to use some closedness qualification condition [2] in that case.

In the second part of the talk, an idea for solving the simple bilevel optimization problem will be given. Basis for this algorithm is a penalization of the objective function of the inner problem using the objective function of the outer one. The algorithm computes a sequence of η_k-optimal solutions of minimizing a Moreau-Yosida regularization of this function over the feasible set of the inner problem. It can be shown that the sequence of computed iterates converges to a solution of the simple bilevel optimization problem provided this problem has a solution.

References

[1] S. Dempe and N. Dinh and J. Dutta, “Optimality Conditions for a Simple Convex Bilevel Programming Problem”, in: Variational Analysis and Generalized Differentiation in Optimization and Control, R.S. Burachik and J.-C. Yao (eds.), Springer, 149–162, 2010.

[2] N. Dinh, T.T.A. Nghia and G. Vallet , “A closedness condition and its applications to DC programs with convex constraints”, Optimization, 59, 541 - 560 (2010).

[3] M.V. Solodov, “An explicit descent method for bilevel convex optimization”, Journal of Convex Analysis, 14, 227 - 237 (2007).

On transversality in variational analysis

Alexander Ioffe

We discuss various aspects of newly developed extension of the classical transversality theory to variational analysis and optimization theory. In particular, this includes interpretaions in transversality terms of some key results (relating to subdifferential calculus, necessary optimality conditions and linear convergence of the method of alternating projections) and also a certain set-valued extension of the Thom-Smale transversality theorem.

First order methods beyond Lipschitz gradient continuity

Marc Teboulle

A central assumption in first order methods (FOM) is to require the differentiable part of an objective function to have a gradient globally Lipschitz continuous, and hence precluding the use of FOM in many important applications. This very restrictive assumption is traditionally circumvents by linesearch approaches and/or quite complex inner loops which are unavoidably distorting the efficiency and the complexity of the given original method.

We introduce a new framework which allows to lift the usual and restrictive uniform Lipschitz continuity of the gradient. This is done through an easy to check convexity condition with respect to a kernel function, which captures all at once, the geometry of the objective/constraints through the Bregman distance paradigm, and naturally translates into a new descent lemma. We prove that the resulting FOM emerging from this approach on the fundamental composite convex optimization model comes with guaranteed complexity estimates, and pointwise global convergence.

The proposed framework lays the ground for many new erspectives, and for tackling broad classes of challenging problems arising in key applications which were until now, considered as out of reach via first order methods. This potential will be demonstrated through the derivation of new and simple schemes with proven efficiency estimates/convergent properties for various important applied models.

This talk is based on a joint work with H.Bauschke and J. Bolte.

Time being permitted, we will present extensions for the nonconvex setting recently obtained in a joint work with J.Bolte, S. Sabach, and Y. Vaisbourd.

Existence, uniqueness, and stability of optimal portfolios of eligible assets

Michel Baes

To protect its clients from the risk of default, any financial institution is required by regulators to hold an adequate capital buffer. The minimal capital buffer the institution needs to raise is not only determined by the regulator’s own risk measure, but also by how this capital buffer is invested once raised: limiting this buffer to a single asset (e.g. cash in only one currency) is generally inefficient as it might lead to higher capital requirements. We are therefore considering multiple eligible assets to raise the capital buffer. This leads us to three fundamental theoretical questions.

1. Do optimal portfolios of several eligible assets, i.e. portfolios that allow to pass the regulatory test at the minimum cost, exist at all?

2. In case optimal portfolios exist, under which conditions are they unique?

3. In case several optimal portfolios exist, how robust is the choice of a specific portfolio? That is, if we base our allocation on a slightly misestimated capital position, how confident can we be that this choice will not be too far for the actual position?

In a first part, we provide a variety of necessary and sufficient conditions for both the existence and the uniqueness of optimal portfolios. The second part is devoted to the analysis of optimal portfolio stability, that we relate to three notions of continuity for point-to-set mapping. We prove that outer semicontinuity is always satisfied. We determine necessary and sufficient for upper semicontinuity. In our financial context, inner semicontinuity proves to be the key stability property. We show how this property may fail even for a convex risk measure, but is satisfied for polyhedral risk measure, such as the Expected Shortfall.

On coercivity of polynomials and real Jacobian conjecture

Tomas Bajbar

We analyze the global diffeomorphism property of polynomial maps by studying the properties of the underlying Newton polytopes at infinity. This allows us to identify a class of polynomial maps for which their global diffeomorphism property is equivalent to their Jacobian determinant being globally non-vanishing. In other words, we identify a class of polynomial maps for which the Real Jacobian Conjecture, which was proven to be false in general, still holds.

Speedup of lexicographic optimization by superiorization and its applications to cancer radiotherapy treatment

Aviv Gibali

Multicriteria optimization problems occur in many real life applications, for example in cancer radiotherapy treatment and in particular in intensity modulated radiation therapy (IMRT). In this talk we focus on optimization problems with multiple objectives that are ranked according to their importance. We solve these problems numerically by combining lexicographic optimization with our recently proposed level set scheme, which yields a sequence of auxiliary convex feasibility problems; solved here via projection methods. The projection enables us to combine the newly introduced superiorization methodology with multicriteria optimization methods to speed up computation while guaranteeing convergence of the optimization.

This is a joint work with Esther Bonacker, Karl-Heinz Küfer and Philipp Süss (Fraunhofer ITWM)

A new projection-type approximation method for solving pseudomonotone variational inequality in Hilbert spaces

Yekini Shehu

Our purpose in this presentation is to introduce a new projection-type approximation method which involves only one projection per iteration for solving variational inequality problem where the underline operator is pseudomonotone and L-Lipschitz-continuous mapping. We establish strong convergence result of the sequence of iterates generated by this method, under mild conditions, in real Hilbert spaces. Our computational overhead is less per iteration than many existing projection-type algorithms for solving this set of problem in the literature and our method is particularly useful when computing projection is the dominating task in the iteration process. Our result generalizes many recent results on variational inequality problem involving monotone mapping. Sound computational experiments comparing our newly proposed method to the existing state of the art on multiple realistic test problems are given.

October 18

ADMM for monotone operators: convergence analysis and rates

Radu Ioan Bot

We discuss a unifying scheme for several algorithms designed to solve monotone inclusion problems involving compositions with linear continuous operators in infinite dimensional Hilbert spaces. We show that a number of primaldual methods for monotone inclusions, and also the classical ADMM numerical scheme for convex optimization problems, along with some of its variants, can be embedded into this unifying scheme. We report on convergence results for the iterates, and on convergence rates obtained by combining variable metric techniques with dynamic step size strategies. The talk relies on common work with E.R. Csetnek.

Tatonnements for Cobb-Douglas economy based on the power method

Vladimir Shikhman

We consider the general economy with consumers maximizing Cobb-Douglas utilities from the algorithmic perspective. It is known that finding equilibrium prices reduces to the eigenvalue problem for a particularly structured stochastic matrix. We show that the power method for solving this eigenvalue problem can be naturally interpreted as a tatonnement executed by an auctioneer. Its linear rate of convergence is established under the reasonable assumption of pairwise connectivity w.r.t. commodities within the submarkets. It is shown that the pairwise connectivity remains valid under sufficiently small perturbations of consumers' tastes and endowments. Moreover, the property of pairwise connectivity holds for almost all Cobb-Douglas economies, i.e. in the regular case. In collaboration with V. Ginsburgh and Yu. Nesterov.

Second-order sufficient optimality conditions for strong local minima in sparse optimal control of reaction diffusion equations

Eduardo Casas and Fredi Tröltzsch

In the numerical solution of sparse optimal control problems for the FitzHugh-Nagumo equations, we observed stability of the optimal state also for very small Tikhonov regularization parameter. This indicated that second order optimality condition should hold for vanishing Tikhonov parameter. Motivated by this observation, we developed second-order optimality conditions that are sufficient for strong local solutions in the sense of calculus of variations. In the talk, these conditions are discussed and some motivating numerical examples are presented. The conditions are applicable to optimal control problems with semi linear parabolic state systems and smooth objectives without Tikhonov regularization term. They can also be used for sparse optimal control problems, where a multiple of the L^1-norm of the control is included in the objective.

Convergence analysis of the Frank-Wolfe algorithm in Banach spaces

Hong-Kun Xu

The Frank-Wolfe algorithm (FWA), also known as the conditional gradient algorithm, was introduced by Marguerite Frank and Philip Wolfe in 1956. Due to its simple linear subproblems, FWA has recently been paid much attention to solve constrained optimization problems over closed convex bounded sets. The convergence of FWA depends on the way of choosing the sequence of stepsizes. In this talk, we will report some recent convergence results on FWA in the Banach space setting by using two ways of choosing the stepsizes: one way is by the one-dimensional line minimization and the other is by the open loop rule. In addition, we will also discuss the sublinear rate of convergence by introducing the concept of curvature constant of order bigger than one, which includes the case where the Fr´echet derivative of the objective function satisfies the H¨older continuity condition, in particular, the Lipschitz continuity condition. The use of Holder condition in analyzing the rate of convergence seems to be the first time in the study of FWA in Banach spaces.

Regular sequences of quasi-nonexpansive operators and their applications

Andrzej Cegielski

We present a systematic study of regular sequences of quasi-nonexpansive operators in Hilbert space. We are interested, in particular, in weakly, boundedly and linearly regular sequences of operators. We show that these types of the regularity is preserved under relaxations, convex combinations and products of operators. Moreover, in this connection, we show that the weak, bounded and linear regularity lead to weak, strong and linear convergence, respectively, of various iterative methods. This applies, in particular, to block iterative and string averaging projection methods, which, in principle, are based on the above-mentioned algebraic operations applied to projections. Finally, we show an application of regular sequences of operators to variational inequality problems.

Numerical methods of optimum experimental design based on a second-order analysis of parameter estimates

Ekaterina Kostina

A successful application of model-based simulation and optimization of dynamic processes requires an exact calibration of the underlying mathematical models. Here, fundamental tasks are the estimation of unknown model coefficients by means of real observations and design of optimal experiments. After an appropriate numerical treatment of the differential systems, the parameters can be estimated as the solution of a finite dimensional nonlinear constrained parameter estimation problem. Due to the fact that the measurements always contain defects, the resulting parameter estimate cannot be seen as an ultimate solution and a sensitivity analysis is required, to quantify the statistical accuracy. The goal of the design of optimal experiments is the identification of those measurement times and experimental conditions, which allow a parameter estimate with a maximized statistical accuracy. Also, the design of optimal experiments problem can be formulated as an optimization problem, where the objective function is given by a suitable quality criterion based on the sensitivity analysis of the parameter estimation problem. Usual choice is a function of a covariance matrix. In this talk we present a new objective function, called Q-criterion, which is based on a second order sensitivity analysis of parameter estimates. The robustness properties of the new objective function in terms of parameter uncertainties is investigated and compared to a worst-case formulation of the design of optimal experiments problem. It is revealed that the Q-criterion covers the worst-case approach of the design of optimal experiments problem based on the A-criterion. Moreover, the properties of the new objective function are considered in several examples. Here, it becomes evident that the Q-criterion leads to a drastic improve of the Gauss-Newton convergence rate for the underlying parameter estimation problems. Based on joint work with M. Nattermann (Bayer AG)

Continuity properties of risk averse stochastic programs

Aloise Pichler

This talk considers stochastic problems, in particular multistage stochastic optimization programs. We address a distance for discretetime, continuous state stochastic processes. The process distance is important in stochastic optimization, as typical stochastic programs are continuous with respect to the process distance. We then generalize the setting to include risk averse stochastic problems. It turns out that important risk measures, which are typically employed in stochastic optimization, are continuous with respect to the distance considered.

Fixed points of Legendre-Fenchel type transforms and polarity type operators

Daniel Reem

A recent result characterizes the fully order reversing operators acting on the class of lower semi continuous proper convex functions in a real Banach space as certain linear deformations of the Legendre-Fenchel transform. Motivated by the Hilbert space version of this result and by the well-known result saying that this convex conjugation transform has a unique fixed point, namely the normalized energy function, we investigate the fixed-point equation in which the involved operator is a fully order reversing operator acting on the above-mentioned class of functions. It turns out that this nonlinear equation is very sensitive to the involved parameters and can have no solution, a unique solution, or several (possibly infinitely many) ones. We also consider a convex geometry version of this equation and obtain a similar characterization of its set of solutions (in particular, the equation is

uniquely solvable by an ellipsoid if a certain key linear operator is positive definite). Our analysis yields a few byproducts, among them ones related to coercive bilinear forms (essentially a quantitative convex analytic converse to the celebrated Lax-Milgram theorem from partial differential equations) and to functional equations and inclusions involving monotone operators.
The talk is based on joint works with Alfredo N. Iusem (IMPA) and Simeon Reich (The Technion).

Lexicographic tangents and facially dual complete cones

Vera Roshchina

We employ the idea of lexicographic tangents (closely related to Nesterov's lexicographic derivatives) to geometric characterisations of the boundary structure of closed convex cones.
In particular, we obtain (different) necessary and sufficient conditions for a cone to be facially dual complete. We also discuss examples of four-dimensional cones whose facial structure exhibits some interesting and somewhat unexpected properties, and show the gap between the necessary and sufficient conditions that we have obtained.
The talk is based on joint work with Prof. Levent Tunçel (University of Waterloo, Canada), arXiv:1704.06368.

Subgradient projection algorithm with computational errors

Alexander J. Zaslavski

We study the subgradient projection algorithm for minimization of convex and nonsmooth functions, under the presence of computational errors. We show that our algorithms generate a good approximate solution, if computational errors are bounded from above by a small positive constant. Moreover, for a known computational error, we find out what an approximate solution can be obtained and how many iterates one needs for this.

October 19

Optimization problems with sparsity-inducing terms

Amir Beck

We consider several optimization problems that contain the l_0-norm expression. These problems are considered to be extremely difficult to solve and analyze due to the nonconvexity and discontinuity of the l_0 expression. We establish under some symmetry assumptions a hierarchy between stationarity-based optimality conditions and conditions based on coordinate-wise optimality. These results also imply a hierarchy between several corresponding algorithms. A key mathematical tool used in analysis is the proximal mapping of symmetric functions including l_0-norm expressions. Joint work with Nadav Hallak.

Finding a best approximation pair of points for two polyhedra

Yair Censor

Given two disjoint convex polyhedra, we look for a best approximation pair relative to them, i.e., a pair of points, one in each polyhedron, attaining the minimum distance between the sets. Cheney and Goldstein showed that alternating projections onto the two sets, starting from an arbitrary point, generate a sequence whose two interlaced subsequences converge to a best approximation pair. We propose a process based on projections onto the half-spaces defining the two polyhedra, which are more negotiable than projections on the polyhedra themselves. A central component in the proposed process is the Halpern-Lions-Wittmann-Bauschke algorithm for approaching the projection of a given point onto a convex set.

This is joint work with Ron Aharoni and Zilin Jiang.

Stationary Point Set: Topological Universality of Convex Quadratic Problems

Harald Günzel

We observe that the jet-space which is used to describe stationary points, Fritz John points and some basic constraint qualifications, is diffeomorphic to the product of state- and parameter-space of a natural (parametric) family of convex quadratic problems.

Since the diffeomorphism used to establish the latter property is just the corresponding jet-extension, this shows the topological universality of the quadratic family, i.e. topological properties of stationary point sets that are generically possible in general parametric optimization problems are already present in convex quadratic optimization.

We can use natural embeddings between stationary point sets and violation sets of the Mangasarian-Fromovitz constraint qualification (of different problem size) to make the universality theorem prove some other topological properties.

Optimal feedback control for a perimeter traffic flow at an urban region

Ilya Ioslovich

Traffic flow control has motivated many researchers since early decades of the 19th century. The concept of a perimeter traffic control for an urban region assumes that a perimeter controller, located at a region border, can manipulate the transfer flows across the border to maximize the total outflow of the region.

The macroscopic fundamental diagram (MFD), that relates average flow with accumulation, is used to model the traffic flow dynamics in the region.

The explicit solution for the optimal feedback control policy and a proof of optimality are provided. The proof is based on the modified Krotov-Bellman sufficient conditions of optimality, and the upper and lower bounds of state variables are calculated.   This is a joint work with Jack Haddad.

Jumps detection in Besov spaces via a new BBM formula. Applications to Aviles-Giga type functional

Arkady Polyakovsky

About 15 years ago, Bourgain, Brezis and Mironescu proposed a new characterization of BV and W^(1,q) spaces (for q>1) using a certain double integral functional involving radial mollifiers.

We study what happens when one changes the power of |x-y| in the denominator of the integrand from q to 1.

It turns out that, for q>1 the corresponding functionals "see" only the jumps of the BV-function. We further identify the function space relevant to the study of these functionals as the appropriate Besov space.

We also present applications to the study of singular perturbation problems of Aviles-Giga type.

A framework for globally convergent methods in nonsmooth and nonconvex problems

Shoham Sabah

Large scale nonsmooth and nonconvex optimization models arise in various data science paradigms. They induce many highly challenging mathematical and computational issues. In this talk, we outline a fairly general theoretical framework to derive globally convergent schemes. We then show how this framework can be successfully applied and adapted to design and analyze novel algorithms for various classes of nonsmooth and nonconvex models, by exploiting friendly structures and data information of the problem at hands. Numerical results will illustrate our findings.

Solution methods for Nash equilibrium problems with mixed-integer variables

Simone Sagratella

A multi-agent system in many practical noncooperative frameworks can be modeled as a Nash equilibrium problem (NEP). Many solution methods have been proposed in the literature to compute solutions, or equilibria, of a NEP (see, e.g., [1]). However, most of them are based on reformulating the NEP as a variational inequality or as a suitable nonlinear system of inequalities. Such reformulations are not relevant if any agent's optimization problem comprises some discrete variables. Indeed, in the resulting mixed-integer setting, in order to solve the NEP, it is necessary to directly refer to the definition itself of equilibrium. With this in mind, we consider wide classes of NEPs with mixed-integer variables that can be provably solved by using suitable Jacobi-type best-response methods. Moreover, we define a branch and bound type algorithm to solve NEPs with mixed-integer variables. Further details on some of these methods can be found in [2, 3, 4].

References

[1] Francisco Facchinei and Jong-Shi Pang. Finite-dimensional variational inequalities and complementarity problems. Springer Science & Business Media, 2007.

[2] Simone Sagratella. Computing all solutions of Nash equilibrium problems with discrete strategy sets. SIAM J Optim, 26(4):2190–2218, 2016.
[3] Simone Sagratella. Algorithms for generalized potential games with mixed-integer variables. Comput Optim Appl, 2017. https://doi.org/10.1007/s10589-017-9927-4.
[4] Simone Sagratella. Computing equilibria of Cournot oligopoly models with mixedinteger quantities. Math Meth Oper Res, 2017. https://doi.org/10.1007/s00186-017- 0599-8.

Minimization of marginal functions in mathematical programming

Martin Knossalla

We consider marginal functions where the lower-level problem is given by a real-valued objective function and a constraint (inequality-type) set-valued mapping. Typically, exact information of the whole subdifferential is not available for intrinsically nonsmooth objective functions such as for marginal functions. Therefore, the semismoothness of the objective function cannot be proved or is even violated. In particular, in these cases standard nonsmooth methods cannot be used. In this talk, we propose a new approach to develop a converging descent method for this class of nonsmooth functions. This approach is based on so-called continuous outer subdifferentials introduced by us. Overall, it is of constructive nature. Within the algorithmic framework the semismoothness is not required.

Linear convergence rate of extrapolated fixed point algorithms

Rafal Zalas

In this talk we present a certain extrapolation technique which we apply to some well-known projection, subgradient projection and other fixed-point algorithms. All of them can be considered within the general string averaging framework. The analytical results show that under certain assumptions, the convergence can be linear, which is known to be the case for the extrapolated simultaneous projection method.

This is joint work with Christian Bargetz, Victor I. Kolobov and Simeon Reich.