Titles and Abstracts

A Hardy type inequality on fractional order Sobolev spaces on the Heisenberg group

Adimurthi
TIFR Centre For Applicable Mathematics, Bangalore

We define the fractional order Sobolev spaces on the Heisenberg Group with weight, and prove optimal imbedding theorems, compactness of the imbedding and the Hardy-Sobolev imbedding.

Semilinear elliptic problems with a Hardy potential

Catherine Bandle
Universität Basel

In this talk we study the positive solutions of the problem

$$\Delta u + \frac{\mu}{\delta(x)^2} u = u^p \qquad \text{in } \Omega,$$

where $p > 0$, $p \neq 1$ and $\mu \in \mathbb{R}$, $\mu \neq 0$ where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain and $\delta(x)$ is the distance of a point $x \in \Omega$ to the boundary. The singularity of the potential forces the solutions either to blow up or to vanish on the boundary. The nonlinearity gives rise to solutions which blow up ($p > 1$) or which have a dead core ($p < 1$).

The interplay between the potential and the nonlinearity leads to interesting structures of the solution sets. In general, the existence can only be guaranteed if μ is smaller than the Hardy constant and larger than a constant depending on p. By means of ODE techniques we give a complete description of the radial solutions in balls. Motivated by this special case we construct solutions in general domains which are governed by the singular potential.

Two perturbation problems concerning the Hardy constant $H_p(\Omega)$

Gerassimos Barbatis
University of Athens

We consider the L^p-Hardy constant $H_p(\Omega)$ associated with a domain $\Omega \subset \mathbb{R}^n$, and we present results about its variation upon perturbation of the domain Ω and the exponent $p > 1$. Concerning the dependence on Ω we present a Fréchet differentiability result and establish a Hadamard-type formula for the corresponding derivatives. We also show a stability result for the minimizer and a stability estimate in terms of the Lebesgue measure of the symmetric difference of domains. Concerning the p-dependence, we discuss various monotonicity, continuity and differentiability results, mostly for non-convex domains in which case the Hardy constant is in general not explicitly known.

This is joint a work with P. D. Lamberti.

Global sharp estimates for a class of pseudodifferential operators

Matania Ben-Artzi
Hebrew University of Jerusalem

Global spacetime estimates are derived for evolution groups generated by a class of pseudodifferential operators . Optimal smoothing is obtained, along with best constants (generalizing the well-known Schrödinger group). The derivation of best constants in the estimates is closely related to similar best constants in Hardy-type inequalities and trace maps.

A priori estimates and ground states of solutions of some Emden-Fowler equations with gradient term

Marie-Françoise Bidaut-Véron
Universit de Tours

Here we consider the nonnegative solutions of equations in a punctured ball $B(0,R)\setminus\{0\} \subset \mathbb{R}^N$ or in $\mathbb{R}^N$, of two types

$$-\Delta u = u^p |\nabla u|^q \tag{1}$$

where essentially $p+q>1$, and

$$-\Delta u = u^p + M|\nabla u|^q \tag{2}$$

where $p, q > 1$ and $q > 1$ and $M \in \mathbb{R}$. We give new a priori estimates on the solutions and their gradient, and Liouville type results. We use Bernstein technique and Osserman's or Gidas-Spruck's type methods. The most interesting cases correspond to $0 \leq q < 1$ for equation (1) and $q = 2p/(p+1)$ for equation (2).

On the subtleties of a $1D$ regularized interpolation problem

Haim Brezis
Technion and Rutgers University

I will consider functionals of the form

$$F(u) = \int_I |u'| \, \mathrm{d}x + \int_I |u-f|^2 \, \mathrm{d}\mu,$$

where $I = (0,1)$, f is a given (decent) function and μ is a given measure. F is well defined in $W^{1,1}$, but need not admit a minimizer there. On the other hand, F is not well defined on the natural larger space BV, especially when the measure μ is singular, e.g. a sum of Dirac masses – a case already of great interest. I will present various ways of "forcing" F to have minimizers, and I will discuss the properties of the corresponding minimizers.

Heat kernel estimates of Schrödinger-type operators

Baptiste Devyver
Technion

We will present estimates for various heat kernels, in the two (related) settings:

(i) Estimates for the heat kernel of (vector-valued) Schrödinger operators.

(ii) Estimates for the gradient of the heat kernel of the Laplacian, in the manifold setting.

Concerning (i), we will present an essentially optimal result, and we will explain how (i) and (ii) are related. We will also present some partial results concerning (ii), that go beyond the customary assumption of non-negativity of the Ricci curvature. We point out that even in the particular case of *scalar* Schrodinger operators, our results concerning (i) are new.

Partly based on a joint work with T. Coulhon (Paris) and A. Sikora (Sydney).

Nonlinear evolution on manifolds: long-time asymptotics and relations with a sublinear elliptic problem

Gabriele Grillo
Politecnico di Milano

We describe recent results on the asymptotic behaviour of solution of the porous medium equation on negatively curved manifolds. Detailed asymptotics, in terms of curvature bounds, are given. If the curvature is very negative, i.e. if it diverges to minus infinity faster than quadratically, it is shown that solutions approach, in a suitable sense, separable solution constructed in terms of an the minimal solution to an auxiliary elliptic problem, that has independent interest and is in the spirit of previous results of Brezis and Kamin for weighted, Euclidean sublinear equations.

We report on joint works with M. Muratori and J. L. Vázquez.

Absence of eigenvalues of Schrödinger operators with complex potentials

David Krejčiřík
Czech Technical University

We prove that the spectrum of Schrödinger operators in three dimensions is purely continuous and coincides with the non-negative semiaxis for all potentials satisfying a form-subordinate smallness condition. By developing the method of multipliers, we also establish the absence of point spectrum

for electromagnetic Schrödinger operators in all dimensions under various alternative hypotheses, still allowing complex-valued potentials with critical singularities.

This is joint work with Luca Fanelli and Luis Vega.

L^p Hardy inequality on $C^{1,\gamma}$ domains

Pier Domenico Lamberti
Università di Padova

We consider the L^p Hardy inequality involving the distance to the boundary of a domain in the n-dimensional Euclidean space with nonempty compact boundary. We extend the validity of known existence and non-existence results, as well as the appropriate tight decay estimates for the corresponding minimizers, from the case of domains of class C^2 to the case of domains of class $C^{1,\gamma}$ with $\gamma \in (0,1]$. We consider both bounded and exterior domains. The upper and lower estimates for the minimizers in the case of exterior domains and the corresponding related non-existence result seem to be new even for C^2-domains.

This is joint work with Yehuda Pinchover.

Derivative estimates for solutions of divergence form equations with discontinuous coefficients

Yanyan Li
Rutgers University

In this talk we give derivative estimates for solutions of divergence form elliptic equations with piecewise smooth coefficients. The equations arise from the study of composite material. The novelty of these estimates is that, even though they depend on the shape and on the size of the surfaces of discontinuity of the coefficients, they are independent of the distance between these surfaces.

The Gaussian Double-Bubble Conjecture

Emanuel Milman
Technion

The classical Gaussian isoperimetric inequality, established in the 70's independently by Sudakov-Tsirelson and Borell, states that the optimal way to decompose $\mathbb{R}^n$ into two sets of prescribed Gaussian measure, so that the (Gaussian) measure of their interface is minimal, is by using two complementing half-planes. This inequality yields numerous analytic and probabilistic consequences, such as the log-Sobolev inequality for the Gaussian measure, hypercontractivity of the Ornstein-Uhlenbeck semi-group, Gaussian concentration, etc...

A natural generalization is to decompose $\mathbb{R}^n$ into k sets of prescribed Gaussian measure. It is conjectured that when $k \leq n+1$, the configuration whose interface has minimal (Gaussian) measure is given by the Voronoi cells of a (translated) regular $(k-1)$-dimensional simplex. For instance, in the plane ($n = 2$), the interface is conjectured to be a "tripod" or "Y" - three rays meeting at a single point in 120 degree angles.

We confirm this conjecture for $k = 3$ (a.k.a. the Gaussian Double-Bubble Conjecture) for all $n \geq 2$. None of the numerous methods discovered over the years for establishing the classical $k = 2$ case seem amenable to the $k = 3$ case, and our method consists of establishing a Partial Differential Inequality satisfied by the isoperimetric profile.

Time permitting, we will also discuss uniqueness of minimizers and extensions to $k > 3$.

This is joint work (still in progress) with Joe Neeman (UT Austin).

The role of Hardy's inequality in the theory of function spaces

Petru Mironescu
Université Lyon 1

We illustrate the crucial role of Hardy's inequality in the theory of function spaces through two basic examples: the theory of weighted Sobolev spaces and the functional calculus in Sobolev (and Besov) spaces. As more involved applications, we present the factorization of unimodular Sobolev maps and the lifting of unimodular maps in Besov spaces.

Thomas-Fermi type models of external charge screening in graphene

Vitaly Moroz
Swansea University

Graphene is a recently discovered material which consists of exactly one atomic layer of carbon. We discuss density functional theories of Thomas-Fermi and Thomas-Fermi-von Weizsacker type which describe the response of a single layer of graphene to an external electric charge. Mathematically, this amounts to the analysis of two nonlocal variational problems which involve Coulombic terms and a Hardy type potential. We formulate variational setting in which the proposed energy functionals admit minimizers. The associated Euler-Lagrange equations are also obtained, and uniqueness and regularity of the minimizers are proved under some conditions. In addition, we discuss the decay rate (screening) of the minimizers and describe several open problems.

The uncharted territory of $W^{\alpha,1}$ Sobolev spaces

Augusto Ponce
Université catholique de Louvain

I will address properties of the fractional Sobolev space $W^{\alpha,1}(\mathbb{R}^N)$ which cannot be answered by classical representation formulas from Harmonic Analysis, for any exponent $\alpha > 0$. They can be handled instead in terms of a strong capacitary inequality which is based itself on a new geometric boxing inequality that connects the Hausdorff content of dimension $N - \alpha$ and the fractional perimeter of order $0 < \alpha < 1$.

These results have been obtained in collaboration with D. Spector (National Chiao Tung University, Taiwan).

Heteroclinic solutions of mountain pass type

Paul Rabinowitz
University of Wisconsin

Consider the Hamiltonian system of ordinary differential equations:

$$q'' + V_q(t, q) = 0, \qquad \text{for } q \in \mathbb{R}^m, \tag{HS}$$

where V is 1-periodic in t and a double well potential in q. Under mild additional conditions on V, it is known that there is a solution, Q, of (HS) that connects the minima of V, i.e. Q is a heteroclinic solution of (HS). Recently an analogous result was proved for an elliptic system of PDEs in a cylindrical domain in $\mathbb{R}^n$ under Neumann boundary conditions. The above solutions are all obtained by minimization arguments. Our goal here is to describe how to find additional heteroclinic and homoclinic solutions of mountain pass type for (HS) and the elliptic system.

Small energy Ginzburg-Landau minimizers in $\mathbb{R}^3$

Itai Shafrir
Technion

We prove that a local minimizer of the Ginzburg-Landau energy in $\mathbb{R}^3$ satisfying the condition

$$\liminf_{R\to\infty} \frac{E(u; B_R)}{R \ln R} < 2\pi$$

must be constant. The main tool is a new sharp η-ellipticity result for minimizers in dimension three that might be of independent interest.

This is a joint work with Etienne Sandier (Paris 12).

Very singular and large solutions of quasilinear parabolic equations with degenerate absorption

Andrey E. Shishkov
NAS of Ukraine, and RUDN University, Moscow

We study propagation of strong singularities of generalized solutions of nonlinear diffusion–absorption type equations with degenerating on some manifold $\Gamma \subset [0, T] \times \Omega$, $T < \infty$, $\Omega \in \mathbb{R}^n$, $n \geqslant 1$, absorption potential $b(t, x) \geqslant 0$.

In the case when $\Gamma_0 := \Gamma \cap \{0\} \times \Omega \neq \emptyset$ there was elaborated method (see [1], [2]) which allowed to establish sharp conditions on asymptotic of $b(t, x)$ near to Γ, guaranteeing propagation or nonpropagation of strong singularities of solution from Γ_0 onto Γ.

A new version of local energy method gave possibility to treat the case of final degeneracy: $\Gamma \subset \{T\} \times \overline{\Omega}$, $\Gamma \cap \partial\Omega \neq \emptyset$. Particularly, for arbitrary large solution u ($u\big|_{(0,T)\times\partial\Omega} = \infty$) there are obtained sharp condition of propagation or nonpropagation of singularity along Γ. Moreover, in the case of regional blow-up there are obtained sharp upper estimate of final profile of solution as $t \to T$ (see, preliminary results in [3], [4]).

References

[1] Shishkov A., Veron L. Singular solutions of some nonlinear parabolic equations with shatially in homogeneous absorption, *Calc. Var. Part. Differ. Equat.*, **33** (2008), 343–375.

[2] Marcus M., Shishkov A. Propagation of strong singularities in semilinear parabolic equations with degenerate absorption, *Ann. Sc. Norm. Super. Pisa Cl. Sci (5)*, **16** (2016), 1019–1047.

[3] Du Y., Peng R., Polaĉik P. The parabolic logistic equation with blow-up initial and boundary values, *Journal D'Analyse Mathematique* **118** (2012), 297–316.

[4] Shishkov A. Large solutions of parabolic logistic equation with spatial and temporal degeneracies, *Disc. Contin. Dynam. Syst. Series S*, **10** (2017), 895–907.

A class of elliptic systems in the study of non abelian Chern-Simons vortices

Gabriella Tarantello
Seconda Università di Roma

We shall discuss the solvability of a class of elliptic systems involving Dirac sources arising in the study of non-abelian Chern-Simons vortices. Different boundary conditions will be considered in order to account for the so-called "topological" and "non-topological" vortices.

Best constants in Hardy and Hardy Sobolev inequalities: The role of Geometry

Achilles Tertikas
University of Crete

Defect of compactness in Sobolev embeddings on Riemannian manifolds

Cyril Tintarev
Uppsala University

We consider embedding $H^{1,2}(M) \hookrightarrow L^p(M)$, $2 < p < \frac{2N}{N-2}$, where M is a smooth N-dimensional non-compact Riemannian manifold of bounded geometry.

This embedding is not compact. By defect of compactness we understand the difference $u_k - u$ between a weakly convergent sequence in $H^{1,2}(M)$ $u_k \rightharpoonup u$ modulo a remainder vanishing in $L^p(M)$. When M has a rich isometry group, defect of compactness can be expressed, for a suitable subsequence, in the form $\sum_{n\in\mathbb{N}} w^{(n)} \circ g_k^{(n)}$, where $g_k^{(n)}$ are isometries on M and *elementary concentations* $w^{(n)} \circ g_k^{(n)}$ have asymptotically disjoint supports. Functions $w^{(n)}$ are called *concentration profiles* and are obtained as weak limits of $u_k \circ [g_k^{(n)}]^{-1}$.

We show that in the case of general M one can also express defect of compactness as a sum of elementary concentrations, but these are now based on the behavior of M at infinity. Any discrete sequence (y_k) on M allows to present a set of gluing data with transition functions given by $\lim_{k\to\infty} \exp_{y'_k}^{-1} \exp_{y_k}$. From this data one patches a manifold M_∞ associated with the sequence (y_k), with naturally induced Riemannian metric, and on this manifold one patches local profiles defined as weak limits of $u_k \circ \exp_{y_{k,i}}$ into a global profile $w \in H^{1,2}(M)$. An elementary concentration associated with the sequence $(y_k)$is then patched on $w_i \circ \exp_{y_{k,i}}^{-1}$ on M. The following identities are satisfied:

$$\|u_k\|_{1,2}^2 \to \|u\|_{1,2}^2 + \sum_{n\in\mathbb{N}} \|w^{(n)}\|_{1,2}^2 \,,$$

and

$$\int_M |u_k|^p d\mu = \int_M |u|^p d\mu + \sum_{n\in\mathbb{N}} \int_{M_\infty^{(n)}} |w^{(n)}|^p d\mu_\infty^{(n)} + o(1).$$

This is a joint work with Leszek Skrzypczak

Nonlinear elliptic and parabolic equations driven by fractional operators

Juan Luis Vázquez
Universidad Autónoma Madrid

The talk presents work on the existence and bahaviour of solutions of nonlinear fractional elliptic and parabolic equations, mainly when posed in bounded domains. Attention is given to functional aspects, to the boundary behaviour and the long time asymptotics.

Recent works are collaboration with M. Bonforte, A. Figalli and Y. Sire.

General reference: J. L Vázquez, The mathematical theories of diffusion. Nonlinear and fractional diffusion, in "Nonlocal and Nonlinear Diffusions and Interactions: New Methods and Directions", Springer Lecture Notes in Mathematics, C.I.M.E. Foundation Subseries.

Sublinear equations and related weighted norm inequalities

Igor E. Verbitsky
University of Missouri, Columbia

Global pointwise estimates of solutions will be presented for sublinear elliptic equations and their quasilinear analogues with measure coefficients and data. Similar results for nonlocal operators of the fractional Laplacian type, as well as integral operators with positive kernels satisfying various forms of the maximum principle, will be discussed, along with associated weighted norm inequalities. This talk is based on joint work with Alexander Grigor'yan, Stephen Quinn, and Adisak Seesanea.

Initial trace of positive solutions to fractional diffusion equation with absorption

Laurent Véron
Université de Tours

We prove the existence of an initial trace $\mathcal{T}_u$ of a positive solution u to the semilinear fractional diffusion equation (H)

$$\partial_t u + (-\Delta)^\alpha u + f(t,x,u) = 0 \quad \text{in} \quad \mathbb{R}_+^* \times \mathbb{R}^N, \tag{H}$$

where $N \geq 1$, the operator $(-\Delta)^\alpha$ with $\alpha \in (0,1)$ is the fractional Laplacian, and $f : \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R} \mapsto \mathbb{R}$ is a Caratheodory function satisfying $f(t,x,u)u \geq 0$ for all $(t,x,u) \in \mathbb{R}_+ \times \mathbb{R}^N \times \mathbb{R}$. We define the regular set of the trace $\mathcal{T}_u$ as an open subset of $\mathcal{R}_u \subset \mathbb{R}^N$ carrying a nonnegative Radon measure ν_u such that

$$\lim_{t\to 0} \int_{\mathcal{R}_u} u(t,x)\zeta(x)\mathrm{d}x = \int_{\mathcal{R}_u} \zeta \mathrm{d}\nu \qquad \forall \zeta \in C_0^2(\mathcal{R}_u),$$

and the singular set $\mathcal{S}_u = \mathbb{R}^N \setminus \mathcal{R}_u$ as the set points a such that

$$\limsup_{t\to 0} \int_{B_\rho(a)} u(t,x)\mathrm{d}x = \infty \qquad \forall \rho > 0.$$

We give several estimates of solutions and study the reverse problem of constructing a positive solution to (H) with a given initial trace $(\mathcal{S},\nu)$ where $\mathcal{S} \subset \mathbb{R}^N$ is a closed set and ν is a positive Radon measure on $\mathcal{R} = \mathbb{R}^N \setminus \mathcal{S}$. We develop the study of the model case $f(t,x,u) = t^\beta u^p$ where $\beta > -1$ and $p > 1$.

This is a joint work with Huyuan Chen, Jiangxi Normal University, Nanchang, RPC.