Graphs and Groups

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For a group G and a subgroup $H \leq G$, we say that an element $g \in G$ commensurates H if $H \cap gHg^{-1}$ has finite index in H and in gHg^{-1}. We define the commensurator of H in G to be the subgroup

$$\mathrm{Comm}_G(H) = \{g \in G \mid g \text{ commensurates } H\}.$$

Let T_d be the d-regular tree ($d \geq 3$), and let $\mathrm{Aut}(T_d)$ be the group of all automorphisms of T_d. Let $\Gamma \leq \mathrm{Aut}(T_d)$ be a uniform lattice, i.e a subgroup Γ of the group of all automorphisms $\mathrm{Aut}(T_d)$ that acts on T_d such that the stabilizers of vertices are finite and there are finitely many orbits of vertices.

In this project we will study the group $C = \mathrm{Comm}_{\mathrm{Aut}(T)}(\Gamma)$. It turns out that this group does not depend on the choice of the uniform lattice Γ, and its elements have a nice description using finite colored graphs. It also has a nice (infinite) generating set which we will discuss in the project.

Here are possible questions we will be interested in:

1. What can be said about the structure of elements in C?
2. Which groups appear as subgroups of C?
3. Does C act nicely on some interesting spaces?
4. Is C simple? (this was conjectured by Lubotzky-Mozes-Zimmer)

We will also discuss the connection between elements in C to groups that act on products of 2 trees.

The project does not require any knowledge beyond the introductory courses in group theory and graph theory. Some basic programming can be useful if we choose to run some experiments.