Graphs and Groups

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For a group $G$ and a subgroup $H \leq G$, we say that an element $g \in G$ commensurates $H$ if $H \cap gHg^{-1}$ has finite index in $H$ and in $gHg^{-1}$. We define the commensurator of $H$ in $G$ to be the subgroup

$$\text{Comm}_G(H) = \{g \in G \mid g \text{ commensurates } H\}.$$ 

Let $T_d$ be the $d$-regular tree ($d \geq 3$), and let Aut($T_d$) be the group of all automorphisms of $T_d$. Let $\Gamma \leq \text{Aut}(T_d)$ be a uniform lattice, i.e a subgroup $\Gamma$ of the group of all automorphisms $\text{Aut}(T_d)$ that acts on $T_d$ such that the stabilizers of vertices are finite and there are finitely many orbits of vertices.

In this project we will study the group $C = \text{Comm}_{\text{Aut}(T)}(\Gamma)$. It turns out that this group does not depend on the choice of the uniform lattice $\Gamma$, and its elements have a nice description using finite colored graphs. It also has a nice (infinite) generating set which we will discuss in the project.

Here are possible questions we will be interested in:

1. What can be said about the structure of elements in $C$?
2. Which groups appear as subgroups of $C$?
3. Does $C$ act nicely on some interesting spaces?
4. Is $C$ simple? (this was conjectured by Lubotzky-Mozes-Zimmer)

We will also discuss the connection between elements in $C$ to groups that act on products of 2 trees.

The project does not require any knowledge beyond the introductory courses in group theory and graph theory. Some basic programming can be useful if we choose to run some experiments.