**GROUPS, QUASI-ISOMETRIES, GROWTH, AND HYPERBOLICITY**

RADHIKA GUPTA, MICHAH SAGEEV, EMILY STARK, DANIEL J WOODHOUSE

Students participating in this project will encounter first-hand tools of modern geometry and group theory. A *group* is a mathematical object whose elements correspond to symmetries of a space. For example, a dihedral group corresponds to the symmetries of a polygon, and the integers are a set of symmetries of the line. In fact, every group can be viewed as the set of symmetries of some (possibly infinite) connected graph called the *Cayley graph* for the group. Viewing edges in the graph as paths of length one turns the graph into a metric space. Examples are drawn below. A far-reaching problem in mathematics aims to understand the geometry of these spaces and how the geometry relates to the group.

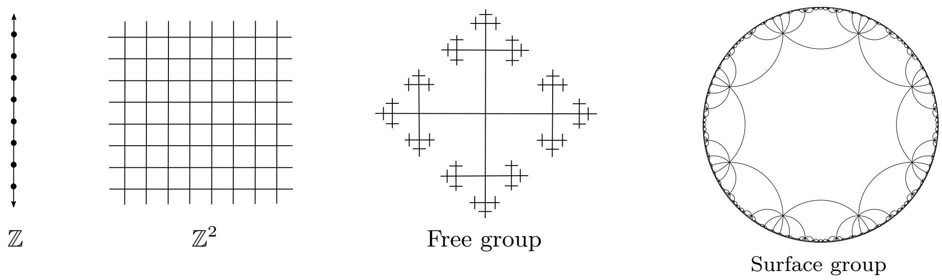


FIGURE 1. Examples of Cayley graphs for specified groups. Here, the tree is a 4-valent regular tree in which each edge has length one, and the octagon tiling is drawn in the disk model of the hyperbolic plane using Curt McMullen's *lim* program.

To begin, we note that Cayley graphs are different than Euclidean spaces and manifolds: the geometry of these graphs is far more *coarse*. So, to study these spaces, we consider *quasi-isometries* between them; these maps are allowed to disturb distances, but only up to bounded multiplicative and additive errors. The advantage of a quasi-isometry is that the large-scale geometry of a Cayley graph can be studied, while the small scale can be ignored.

Many intuitive geometric concepts are invariant under quasi-isometries. A wonderful thing is that the coarse geometry of a group is directly related to algebra of the group. For example, one geometric property is called the *growth* of a group, which is the rate at which the ball of size $n$ in the Cayley graph for the group grows. If $G$ is an infinite group, then the growth of $G$ is at least linear (the integers for example), and is at most exponential (the free group for example). We will see that growth is a quasi-isometry invariant. In fact, in a celebrated theorem, Gromov proved that if $G$ has polynomial growth, then $G$ is virtually nilpotent. We may work to study growth in some new examples. Another notion of coarse geometry with substantial algebraic consequences is called *hyperbolicity.* We will study some hyperbolic spaces and may work to classify them up to quasi-isometry.

**Pre-requisites.** The material will be accessible, requiring only basic understanding of group theory. Students will also benefit from any topology and geometry that they have already studied. This project will be conducted in English.