Mini-course descriptions and abstracts of outreach lectures
Applied Math Summer school on "The complex Math of the real world"
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Mini-course lecture series

Mathematics of fluids in motion

Eduard Feireisl
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Institute of Mathematics of the Czechosloval Academy of Sciences

The goal of this lecture series is to show examples of "synergy" effect between mathematical analysis and numerics in fluid dynamics. We show how certain purely theoretical results may shed some light on convergence of some numerical schemes. Then we undertake a short excursion in the mathematical theory, highlighting some recent rather negative results obtained via the method of convex integration.

Energetic Variational Approaches in Complex Materials

Chun Liu
Department of Applied Mathematics
Illinois Institute of Technology

In these talks, I will discuss the general framework of energetic variational approaches in deriving mean field dynamical models for complex fluids. I will explore various phase field models related to different interfacial motions. In particular, I will introduce various surface contributions and thermal effects.

Optimal transportation between unequal dimensions

Robert McCann
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University of Toronto

In the last few decades, the theory of optimal transportation has blossomed into a powerful tool for exploring applications both within and outside mathematics Its impact is felt in such far flung areas as geometry, analysis, dynamics, partial differential equations, economics, machine learning, weather prediction, and computer vision. The basic problem is to transport one probability density onto other, while minimizing a given cost c(x,y) per unit transported. In the vast majority of applications, the probability densities live on spaces with the same (finite) dimension. In this lecture series we will surveying a few highlights from this theory, before focusing our attention on what can be said when the densities instead live on spaces with two different (yet finite) dimensions. Although the answer can still be characterized as the solution to a fully nonlinear differential equation, it now becomes badly nonlocal in general. Remarkably however, one can identify conditions under which the equation becomes local, elliptic, and amenable to further analysis.

Synchronization

Arkady Pikovsky
Department of Physics
Universität Potsdam

First recognized in 1665 by Christiaan Hyugens, synchronization phenomena are abundant in science, nature, engineering, and social life. Systems as diverse as clocks, singing crickets, cardiac pacemakers, firing neurons, and applauding audience exhibit a tendency to operate in synchrony. These phenomena are universal and can be understood within a common framework based on modern nonlinear dynamics.

Course plan: General introduction, synchronization phenomena in nature and experiments.Basics of nonlinear oscillations, phase reduction, synchronization by periodic forcing and circle map, mutual synchronization. Populations of oscillators, Kuramoto model, Watanabe-Strogatz and Ott-Antonsen theories. Partial synchronization. Oscillatory media. Effects of noise.

Outreach lectures

Towards Understanding Complex Spatio-Temporal Systems

Miro Kramar
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INRIA Saclay

Nonlinear dynamical systems play an important role in modelling of various processes in fields ranging from physics, chemistry and biology to many other natural and social sciences. Despite the importance of nonlinear models and intense efforts of many researchers, the global dynamics of many of these systems is still far from being properly understood. Our comprehension of the dynamics becomes even more tentative if the governing equations are not known. In this case the study of the system is based on data collected from experiments. In this talk I will introduce rigorous mathematical methods for analysing the dynamics of a system from time series and demonstrate these methods on a variety of different problems which exhibit an intricate pattern formation. In the first part I will explain how to describe these patterns using persistent homology. Persistent homology allows us to transform experimental or numerical data into a point cloud in the space of persistence diagrams. There are a variety of metrics that can be imposed on the space of persistence diagrams. By choosing different metrics one can interrogate the pattern locally or globally, which provides deeper insight into the dynamics of the process of pattern formation. In the second part of this talk I will discuss topological methods for identifying robust dynamical structures that act as organising block of the dynamics.

Cellular function given parametric variation: membrane excitability and the Hodgkin-Huxley model

Shimon Marom
Medicine school and Faculty of Electrical Engineering
Technion

Macroscopic cellular function is maintained in spite of extensive variations in underlying elementary constituents, including the size of the cell, and the number, distribution and kinetics of their proteins. I take advantage of the sound theoretical basis of cellular electricity and action potential generation to analyse macroscopic cellular invariance given microscopic variation. This analysis points to a significant gap between the high-dimensional level of description captured by biophysical measurements of channel function and the lower, physiological dimensionality, to which cellular function is sensitive. When examined in a lower dimension, a simple rule that relies on ionic channel diffusion in protein configuration space provides a powerful control mechanism that maintains excitability amid parametric variation.

Extinction of Established Population: A View from Physics

Baruch Meerson
Racah Institute of Physics
Hebrew University of Jerusalem

Extinction of a long-lived self-regulating population is a dramatic effect. It can result from a rare large fluctuation coming from the discreteness of individuals and stochasticity of their births, deaths and interactions. It also epitomizes the importance of rare events. Predicting the mean time to extinction is important in many applications. Examples include assessment of viability of small populations and evaluation of the lifetime of an infectious disease in a closed community. I will show how one can use a variant of WKB approximation, which originates in quantum-mechanics and other areas of physics, to evaluate the mean time to extinction of a stochastic population. In the WKB framework the most likely path of the population to extinction is described by a special trajectory of an underlying classical Hamiltonian system.

Soft nematic solids: textures, defects, and deformations

Len Pismen
Faculty of Chemical Engineering
Technion

Liquid crystal elastomers, made of cross-linked polymeric chains with embedded mesogenic structures, combine orientational properties of liquid crystals with the shear strength of solids. Their flexibility and sensitivity to chemical and physical signals comes close to that of biological tissues. A variety of three-dimensional forms can arise following a phase transition in elastomeric textures. Transitions to a deformed polarized state may be frustrated in constrained geometry leading to the formation of defects. Phase separation and a change of topology of nematic textures may take place due to the coupling between gradients of the nematic order and chemical composition. Reversible local phase transitions causing repeated reshaping can be used to construct soft crawling and swimming robots with the gait and speed dependent on flexural rigidity and substrate friction.

Dynamical Systems Approach to Debris Mitigation and Remediation

Aaron Rosengren
Aerospace and Mechanical Engineering
The University of Arizona

This work focuses on the evolution of satellites and space debris as dynamical systems with the overall intent being to identify and study the different life cycles which these bodies go through. Recent advances in our understanding of the dynamical processes that act on these artificial celestial bodies predict that they may undergo significant chaotic drifts in circumterrestrial phase space throughout their existence. We study the implications of coupled gravitational and non-gravitational perturbations on their long-term orbital dynamics, and show how secular and semi-secular resonances can profoundly affect the behavior of these bodies, in both dissipative and Hamiltonian settings. In this talk, we will present the most complete to date dynamical atlas of the entire usable circumterrestrial space, characterizing the long-term evolution of Earth satellites from LEO to GEO and beyond, for the purposes of passive debris mitigation and remediation. In some regions, the overlapping of the predominant resonances furnish a number of interesting disposal hatches at moderate to low eccentricity orbits. For satellites equipped with an on-board, area-augmenting device to increase their area-to-mass ratios at the end of life, solar radiation pressure was found to generally promote the deorbiting process, particularly at the transition region between LEO and MEO. Here we will link these cartographic stability maps to the appropriate disposal strategy or deorbiting device for any operational orbit, and we highlight in particular how such dynamical assessments can have a profound and tangible influence on space debris remediation though the passive debris removal ideology. Although direct reentry from very low eccentricities is very unlikely in most cases of interest, we find that a modest delta-v budget would be enough for satellites to be steered into a relatively short-lived resonance and achieve reentry into the Earths atmosphere within realistic timescales. The solution to the debris proliferation problem throughout all space regions can only be found by coupling a deep understanding of the dynamical environments occupied by artificial satellites and space debris with technical, political, and legal solutions.

Emergent behavior in self-organized dynamics: from consensus to hydrodynamic flocking

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Center for Scientific Computation and Mathematical Modeling
University of Maryland

A fascinating aspect in collective dynamics is self-organization: ants form colonies, birds flock, mobile networks coordinate a rendezvous and human crowds reach a consensus. We discuss the large-time, large-crowd behavior of different models for collective dynamics. The models are driven by different rules of engagement which quantify how each member of the crowd interacts with its immediate neighbors.

We address two related questions.

1. How short-range interactions lead, over time, to the emergence of long-range patterns;
2. How the flocking behavior of large crowds is captured by their hydrodynamic description.

Here we introduce a new communication protocol which incorporates short-range kernels restricted to geometric balls and adapted to topological neighborhoods dictated by the local density inside these balls. We prove the unconditional emergence of flocking.