Classifying critical points of nonconvex quadratic functions

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We consider smooth unconstrained optimization problem of the form

$$\min\left\{f(x):\ x\in\mathbb{R}^d\right\},$$

where $f:\mathbb{R}^d\to\mathbb{R}$ is a differentiable function. This problem is solved using iterative algorithms that converge to critical points of the problem, that is, points which satisfy $\nabla f(x)=0$. Critical points that are not local minimizers are of little interest in the context of optimization problems, so a desirable property of such algorithms is to avoid such points.

Instead of treating a general non-convex function, in this project, we focus on the case where f is a nonconvex quadratic function. Optimization problems with nonconvex quadratic objective appear in a variety of important applications in signal processing and data analysis, to mention just a few. Focusing on these problems facilitates a systematic approach to the problem of characterizing the critical points. Indeed, even under these "nice" structure, characterizing critical points and identifying properties of an algorithm which enable convergence to a local minimum with low objective value is both necessary and challenging.

Prerequisites: To participate in this project the student should have taken the first-year courses in calculus and linear algebra. Programming skills are also an asset.