The Lie algebra of modular knots

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In the eighties, Goldman invented a way to define the bracket for two homotopy classes of loops on a surface, as in the following figure (see [2]):

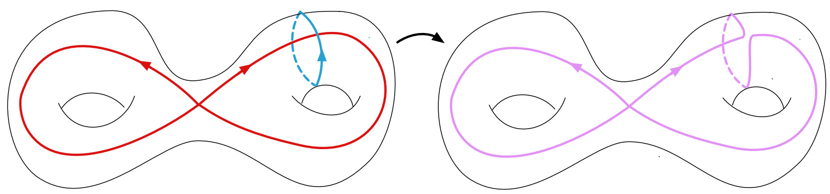

FIGURE 1. The Lie bracket of two loops.

This makes the space of loops into a Lie algebra. There are several interesting open problems regarding the brackets, and several generalizations. For example there's a generalization for curves in a three dimensional manifolds, due to Chas and Sullivan. The problem here is that curves in three manifolds generically do not intersect. Thus, the bracket has to be defined differently (and they define it for families of curves).

In this project we will consider specifically loops on the modular surface $M = \mathbb{H}^2/\mathbf{PSL}_2(\mathbb{Z})$, which is simply the space of all lattices in the plane up to scaling and rotations.

We will use a very nice way found by Ghys to canonically lift these curves to curves in S^3 (reviewed in [1]). Therefore, we can try and answer the following problem suggested by Moira Chas:

Question. Can we understand the geometric action of the Goldman bracket on the lifted modular loops?

Hopefully, an answer to this will give clues for defining the bracket for general curves in the relevant subspace of S^3, or maybe even in more general three dimensional manifolds.

Background: Some background in topology (for instance knowing what a fundamental group is) will be assumed. There is no need to know anything about Lie algebras, and familiarity with the hyperbolic plane or the group $\mathbf{PSL}_2(\mathbb{Z})$ will be an advantage but is not essential.

We will mainly follow the following two sources, and it will be great if you can look at [2] in advance.

[1] Alex Brandts, Tali Pinsky, and Lior Silberman. Volumes of hyperbolic three-manifolds associated to modular links. *arXiv preprint arXiv:1705.04760*, 2017.

[2] Moira Chas. The Goldman bracket and the intersection of curves on surfaces. In *Geometry, groups and dynamics*, volume 639 of *Contemp. Math.*, pages 73–83. Amer. Math. Soc., Providence, RI, 2015.