The H-theorem for the nonlinear Fokker-Planck equation

Viorel Barbu

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One proves the existence of a Lyapunov function for the nondegenerate Fokker-Planck equation, $u_t - \Delta(\beta(u)) + \mathrm{div}(Db(u)u) = 0$, and convergence in $L^1(\mathbb{R}^d)$ of the solution $u(t)$ to a stationary solution as t goes to infinity.

Predators-prey model with competition

Henri Berestycki

EHESS, PSL University, Paris, France

I report on joint work with Alessandro Zilio about a new model for predators like wolves that can divide into several hostile packs. We aim at showing how territories and packs are formed among certain predators interacting with prey. This model, an extension of the original Lotka-Volterra system, rests on basic principles of predator-prey interaction and competition.

We analyze stationary states and various asymptotic behaviors of this system, especially when the competition parameter becomes unbounded. The questions we address are to understand the conditions under which predators segregate into packs, how many packs a given environment can sustain, and whether there is an advantage to have such hostile packs.

Some results for the vectorial Allen-Cahn equation

Fabrice Bethuel

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Whereas the scalar Allen-Cahn equation is rather well under stood, I will discuss several issues for the vectorial case: Concentration on codimensional-one sets, construction of profiles or pseudo-profiles, and links with monotonicty formulae.

Some (perhaps) surprising results on nonlinear Dirichlet problems

Lucio Boccardo

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Here we point out two of the results I will present in the talk.
Let Ω be a bounded, open subset of $\mathbb{R}^N$, $N \geq 2$, $p > 1$. We consider nonlinear differential operators of the type $A(v) = -\mathrm{div}\,(a(x, \nabla v))$, under the classical assumptions on $a(x, \xi)$ (Carathéodory function, such that, for almost every $x \in \Omega$, for every $\xi \in \mathbb{R}^N$ and η in $\mathbb{R}^N$, with $\xi \neq \eta$, $a(x, \xi) \cdot \xi \geq \alpha\,|\xi|^p$, $|a(x, \xi)| \leq \beta\,|\xi|^{p-1}$, $\big(a(x, \xi) - a(x, \eta)\big) \cdot (\xi - \eta) > 0$, where α and β are positive constants).

- It is well known that, in general, it is not possible to pass to the limit, under weak convergence, in nonlinear problems. Indeed, if we deal with the problems

$$w_n \in W_0^{1,p}(\Omega) : \int_\Omega a(x, \nabla w_n)\nabla v = \int_\Omega F_n \, \nabla v, \ \forall v \in W_0^{1,p}(\Omega),$$

with $\{F_n\}$ weakly convergent to F_∞ in $(L^{p'}(\Omega))^N$, in [B-Gallouet, Nonlin.Anal. 2016] a counterexample is given where the sequence $\{F_n\}$ weakly converges to F_∞ in $(L^{p'}(\Omega))^N$, but the sequence $\{w_n\}$ weakly converges to some w^* in $W_0^{1,p}(\Omega)$, with $w^* \neq w_\infty$.
In [L.B. - 75 Haim, preprint] we prove that, if we consider the Dirichlet problems

$$u_n \in W_0^{1,p}(\Omega) : \int_\Omega a(x, \nabla u_n)\nabla v = \int_\Omega f_n(x) \, v(x), \ \forall v \in W_0^{1,p}(\Omega),$$

where $\{f_n\}$ is a sequence weakly convergent to f_∞ in $L^{(p^*)'}(\Omega)$, then it is possible to prove that the sequence $\{u_n\}$ weakly converges in $W_0^{1,p}(\Omega)$ (and no more) to u_∞ .

- In [Arcoya-B, J.Funct.Anal. 2015] we proved the existence of bounded weak solutions in $W_0^{1,p}(\Omega)$ of the Dirichlet problem (with very singular right hand side)

$$\psi \in W_0^{1,p}(\Omega) : \int_\Omega a(x, \nabla \psi)\nabla \psi + \int_\Omega b(x) \, \psi |\psi|^{p-1} = \int_\Omega f(x) \, \psi, \ \forall v \in W_0^{1,p}(\Omega),$$

under the assumption

$$\exists \, Q : |f(x)| \leq \, Q \, b(x) \in L^1(\Omega).$$

On the orthotropic Laplacian

Pierre Bousquet

Université de Toulouse, France

We present some new regularity results for the orthotropic harmonic functions, which are the minimizers of a degenerate and anisotropic variant of the Dirichlet functional.

These results have been obtained in collaboration with L. Brasco (Ferrara), V. Julin (Jyvaskyla), C. Leone (Naples) and A. Verde (Naples).

Thin Film Liquid Crystals with Oblique Anchoring and Boojums

Lia Bronsard

McMaster University, Canada

We study a two-dimensional variational problem which arises as a thin-film limit of the Landau-de Gennes energy of nematic liquid crystals. We impose an oblique angle condition for the nematic director on the boundary, via boundary penalization (weak anchoring.) We show that for strong anchoring strength (relative to the usual Ginzburg-Landau length scale parameter,) defects will occur in the interior, as in the case of strong (Dirichlet) anchoring, but for weaker anchoring strength all defects will occur on the boundary. These defects will each carry a fractional winding number; such boundary defects are known as "boojums". The boojums will occur in ordered pairs along the boundary; for angle $\alpha \in (0, \frac{\pi}{2})$, they serve to reduce the winding of the phase by steps of 2α and $(2\pi - 2\alpha)$ in order to avoid the formation of interior defects. We determine the number and location of the defects via a Renormalized Energy and numerical simulations. This represents joint work with S. Alama and D. Golovaty.

Erratic Extremists Induce Dynamic Consensus: A New Model for Opinion Dynamics

Alfred M. Bruckstein

Technion - Israel Institute of Technology, Israel

A society of agents, with ideological positions, or "opinions" expressed as real values ranging from $-\infty$ (the "far left") to ∞ (the "far right"), is considered. At fixed (unit) time intervals agents repeatedly reconsider and change their opinions if and only if they find themselves at the extremes of the range of ideological positions. Extremist agents are erratic: they become either more radical, and move away from the positions of other agents, with probability ε, or more moderate, and move towards the positions held by peers, with probability $(1 - \varepsilon)$. The change in the opinion of the extremists is one unit on the real line. We prove that the agent positions cluster in time, so that non-extremist agents are eventually located within a unit interval. However, the "consensus opinion" is dynamic. Due to the extremists' erratic behavior the clustered opinion set performs a sluggish random walk on the entire range of possible ideological positions (the real line). The inertia of the group, i.e., the reluctance of the society's agents to change their consensus opinion, increases with the size of the group. The extremists perform biased random walk excursions to the right and left and, in time, their actions succeed to move the society of agents in random directions. The "far left" agent effectively pushes the group consensus toward the right, while the "far right" agent counter-balances the push and causes the consensus to move toward the left. We believe that this model, and some of

its variations, has the potential to explain the real world swings in societal ideologies that we see around us.

(joint work with Dmitry Rabinovich)

Wave Equations arising from Geometry

Sagun Chanillo

Rutgers University, USA

We consider wave equations with quadratic derivative nonlinearities in 2D. We randomize the initial data and show that all Picard iterates exist for short time for randomized data in the energy space. In addition there is quantization, in the sense, that solutions will not exist for initial data with energies above a certain threshold energy level. This is related to the work of Brezis-Coron for the constant mean curvature equation and their characterization of bubbles for CMC as Riemann spheres. This is joint work with Po-lam Yung and also M. Czubak, D. Mendelson, A. Nahmod and G. Staffiliani.

How to use the nonlinearities in order to control systems

Jean-Michel Coron

Université Pierre & Marie Curie - Paris 6, France

A control system is a dynamical system on which one can act thanks to what is called the control. For example, in a car, one can turn the steering wheel, press the accelerator pedal etc. These are the control(s). One of the main problems in control theory is the controllability problem, i.e. to see if, by using some suitable controls depending on time, one can move from a given situation to a given target. We study this problem with a special emphasis on the case where the nonlinearities play a crucial role. In finite dimension iterated Lie brackets are key tools for the controllability problem as shown in particular by the Chow-Rashevski theorem. Lie brackets give also important results for some control systems modeled by means of partial differential equations. However we do not know how to use them for many other control systems modeled by means partial differential equations. We present methods to avoid the use of iterated Lie brackets. We give applications of these methods to the control of various physical control systems (Euler and Navier-Stokes equations of incompressible fluids, 1-D hyperbolic systems, heat equations, shallow water equations, Korteweg-de Vries equations, Schroedinger equations...) and to the stabilization problem, another of the main problems in control theory.

The p(u)-Laplacian problem

Michel Chipot

University of Zurich, Switzerland

We consider the $p-$Laplacian problem with the exponent of the nonlinearity p depending on the solution u. Both situations when $p(u)$ is a local quantity or when $p(u)$ is nonlocal are studied. For the associated boundary-value local problem, we prove the existence of weak solutions by using a singular perturbation technique. We also prove the existence of weak solutions to the nonlocal version of the associated boundary-value problem. The issue of uniqueness for these problems is addressed, in particular by working out the uniqueness for a one dimensional local problem and by showing that the uniqueness is easily lost in the nonlocal case. (Joint work with H.B. de Oliveira).

Stable solutions to semilinear elliptic equations

Alessio Figalli

ETH Zurich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

A non-local characterization of Sobolev spaces and bounded variation functions

Massimo Gobbino

University of Pisa, Italy

We consider the family of non-local and non-convex functionals introduced and studied in a series of papers by Bourgain, Brezis and Nguyen. These functionals are defined through double integrals in which every pair of points contributes according to some interaction law. We describe a new approach to the problem that allows to prove that the Gamma-limit of these functionals is proportional to a suitable multiple of the Sobolev norm or the total variation, depending on the summability exponent. For some special classes of interaction laws, this approach gives the explicit values of the constants, and shows the existence of smooth recovery sequences. The key point is reducing the problem first to dimension one, then to a finite combinatorial rearrangement inequality, and finally to the asymptotic study of a family of multivariable inequalities.

Based on some joint works with C. Antonucci, M. Migliorini and N. Picenni.

Asymptotic Behavior and Symmetry of Singular Solutions to Certain Geometric PDEs

Zheng-Chao Han

Rutgers University, USA

We will discuss a few questions on the asymptotic behavior of singular solutions to certain geometric PDEs. A prototype problem is the singular Yamabe problem. I will make some remarks concerning the growth rate of such singular solutions related to earlier work of Löwner-Nirenberg, Schoen, Delanoë, and Finn-McOwen. I will also report on some results, jointly with Alice Chang and Paul Yang of Princeton University, on the growth rate and symmetry of complete, locally conformally flat metrics on canonical domains of the round sphere with constant Q-curvature. A specific result is

Theorem 1. Any complete, conformal metric g on $\mathbb{S}^n \setminus \mathbb{S}^l$ for $l \leq \frac{n-2}{2}$ satisfying

$$Q_g \equiv 1 \text{ or } 0, \tag{1}$$

and

$$R_g \geq 0, \tag{2}$$

in $\mathbb{S}^n \setminus \mathbb{S}^l$ has to be symmetric with respect to rotations of $\mathbb{S}^n$ which leave $\mathbb{S}^l$ invariant.

This theorem is a corollary of the following

Theorem 2. Let g be a conformal, complete metric on $\Omega \subsetneq \mathbb{S}^n$ such that (1) and (2) hold in Ω. Then for any ball $B \subset\subset \Omega$ in the canonical metric $g_{\mathbb{S}^n}$, the mean curvature of its boundary ∂B in metric g with respect to its inner normal is nonnegative.

Other applications of this Theorem will also be discussed.

Minimality of degree-one Ginzburg-Landau vortex in the unit ball

Radu Ignat

Université de Toulouse, France

In this talk, we will focus on the standard Ginzburg-Landau energy for N-dimensional maps defined in the unit ball that are equal to the identity on the boundary. A special critical point is the so-called degree-one vortex map given by the identity map multiplied with a scalar radial profile. We will prove the minimality of this solution and also discuss about the uniqueness result. This is a joint work with L. Nguyen, V. Slastikov and A. Zarnescu.

Learning invariant structures for shape analysis

Ron Kimmel

Technion - Israel Institute of Technology, Israel

A desired approach for surface classification is to find a compact algebraic representation for each surface that would be similar for objects within the same class and preserve dissimilarities between classes. Self-functional maps was suggested by Halimi and the lecturer as a surface representation that satisfies these properties, translating the geometric problem of surface classification into an algebraic form of classifying matrices. The proposed map transforms a given surface into a universal isometry invariant form defined by a unique matrix. The suggested representation is realized by applying a functional maps framework to map the surface into itself. The idea is to use two different metric spaces of the same surface for which the functional map serves as a signature. As an example, we suggested the regular and the scale invariant metric spaces to construct two families of eigenfunctions. The result is a matrix that encodes the interaction between the eigenfunctions resulted from two different Riemannian manifolds of the same surface. Using this representation, geometric shape similarity is converted into algebraic distances between matrices. This construction can be beautifully related to efforts to migrate geometry into the arena of deep learning, in a sense learning to understand. In fact, the self-functional maps can be thought of as part of a more general functional map perspective injecting the spectral construction into a deep learning methodology, for example the FMNet suggested by Litany et al.

Existence and uniqueness of Green's function to a nonlinear Yamabe problem

Yanyan Li

Rutgers University, USA

For a given finite subset S of a compact Riemannian manifold (M, g) whose Schouten curvature tensor belongs to a given cone, we establish a necessary and sufficient condition for the existence and uniqueness of a conformal metric on $M \setminus S$ such that each point of S corresponds to an asymptotically flat end and that the Schouten tensor of the new conformal metric belongs to the boundary of the given cone. This is a joint work with Luc Nguyen.

A-priori estimates for positive sub- and super-solutions of Schrodinger equations with Hardy type singularities

Moshe Marcus

Technion - Israel Institute of Technology, Israel

Let D be a smooth bounded domain In R^N. We consider equations of the form $(*)L_V u = 0$ where $L_V = \Delta + V$, V a positive function in $C^1(D)$ such as $V = \gamma/\text{dist}^2(x, F)$, F compact subset of ∂D, γ a constant smaller than the Hardy constant relative to V. Such equations have previously been investigated mainly in the cases $F = \partial D$ and F a singleton. We introduce a notion of boundary trace and discuss related a-priori estimates **for positive sub- and super-solutions of (*) for a large class of potentials**. The results are partially based on joint work with Dr. P.T. Nguyen.

Propagation of bistable fronts through a perforated wall

Hiroshi Matano

Meiji University, Japan

We consider a bistable reaction-diffusion equation on $\mathbb{R}^N$ in the presence of an obstacle K, which is a wall of infinite span with periodically arrayed holes. More precisely, K is a closed subset of $\mathbb{R}^N$ with smooth boundary such that its projection onto the x_1-axis is bounded, while it is periodic in the rest of variables $(x_2, \ldots, x_N)$. We assume that $\mathbb{R}^N \setminus K$ is connected. Our goal is to study what happens when a planar traveling front coming from $x_1 = +\infty$ meets the wall K.

We first show that there is clear dichotomy between 'propagation' and 'blocking'. In other words, the traveling front either completely penetrates through the wall or is totally blocked, and that there is no intermediate behavior. This dichotomy result will be proved by what we call a De Giorgi type lemma for an elliptic equation on $\mathbb{R}^N$. Then we will discuss sufficient conditions for blocking, and those for propagation. This is joint work with Henri Berestycki and François Hamel.

Forced quasi-linear differential equations with periodic nonlinearities: old and new results

Jean Mawhin

Université Catholique de Louvain, Belgium

Motivated by the classical question of the periodic solutions of the forced pendulum equation

$$u'' + a \sin u = e(t),$$

the study of the periodic solutions of the corresponding system

$$u'' + \nabla_u F(t, u) = e(t),$$

with F periodic in each component u_j of u, has motivated many contributions since the years 1980. More recently, extensions have been given to quasilinear problems of the form

$$\left(\frac{u'}{\sqrt{1 - \|u'\|^2}} \right)' + \nabla_u F(t, u) = e(t),$$

first considered by Haïm Brezis and the author in 2010, and to the corresponding partial differential problem

$$\operatorname{div} \left(\frac{\nabla u}{\sqrt{1 - \|\nabla u\|^2}} \right) + \nabla_u F(x, u) = e(x)$$

with Neumann boundary conditions. Various multiplicity results have been obtained using various techniques of critical point theory.

On Hardy's and Caffarelli, Kohn, Nirenberg's inequalities

Hoài-Minh Nguyên

EPFL, Lausanne, Switzerland

In this talk, I will present the full range of Hardy's and Caffarelli-Kohn-Nirenberg's inequalities for fractional Sobolev spaces and improvements of Hardy's inequality in the classical setting. The arguments are quite elementary and are mainly based on Poincare's and Sobolev's inequality for annulus; in particular, integration by parts is not used. The motivation for investigating these inequalities is from the study of non-local, non-convex functionals related to Sobolev and BV norms suggested by Haim Brezis. A brief review on the results of these functionals is also presented.

Entire solutions of Allen-Cahn equation in low dimensions

Frank Pacard

École polytechnique, Paris, France

I will report some results concerning the Allen-Cahn equation emphasizing the rle played by minimal surfaces in construction of solutions. I will mostly concentrate on the description of entire solutions of the Allen-Cahn equation in low dimensional Euclidean spaces.

How large can Hardy-weight be?

Yehuda Pinchover

Technion - Israel Institute of Technology, Israel

In the first part of the talk we will discuss the existence of optimal Hardy-type inequalities with 'as large as possible' Hardy-weight for a general second order elliptic operator defined on a noncompact Riemannian manifold, while the second part of the talk will be devoted to a sharp answer to the question: "How large can Hardy-weight be?".

Properties of a free boundary driven by a line of fast diffusion

Jean-Michel Roquejoffre

Université de Toulouse, France

The situation is the following: a line, having a strong diffusion on its own, exchanges mass with the half plane below, supposed to be a reactive medium. A front propagates both on the line and below, and one wishes to describe its shape. This setting was proposed (collaboration with H. Berestycki and L. Rossi) as a model of how biological invasions can be enhanced by transportation networks.

Numerical simulations, due to A.-C. Coulon, reveal an a priori surprising phenomenon: the solution is not monotone in the direction orthogonal to the line. We will try to understand this feature in the particular case of a free boundary problem that can be obtained as a limiting case of the original reaction-diffusion system.

We will discuss various further qualitative properties of the free boundary, such as its shape at infinity, and what happens when the diffusion on the line becomes infinite.
Joint work with L. Caffarelli.

Caustics, Impressionism, and Starbursts

Jacob Rubinstein

Technion - Israel Institute of Technology, Israel

Two independent problems in optics, one in optical design and one in physiological optics will be presented. I shall show that both problems can be understood and solved via appropriate caustic structures.

Duality, atomic decomposition of predual and distance formula for the space of Bourgain - Brezis - Mironescu

Carlo Sbordone

University of Naples "Federico II", Italy

A new function space $B \subset L^1(Q_0)$, $Q_0 =]0,1[^n$ was recently introduced by Bourgain-Brezis-Mironescu based on the seminorm

$$\|f\|_B = \sup_{0<\varepsilon<1} [f]_\varepsilon$$

and its subspace $B_0 \subset B$ defined by the "little-o" condition

$$[f] = \limsup_{\varepsilon \to 0} [f]_\varepsilon = 0$$

where $[f]_\varepsilon$ is defined with a suitable maximization procedure over families $\mathcal{F}_\varepsilon$ of disjoint ε-cubes under the constraint $\#\mathcal{F}_\varepsilon \leq \frac{1}{\varepsilon^{n-1}}$.

In a forthcoming paper with L. D'Onofrio, L. Greco, K.M. Perfekt and R. Schiattarella, some new properties of the pair B_0, B are presented.

References

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Estimates for the Hopf invariant in critical fractional Sobolev spaces

Jean Van Schaftingen

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The Brouwer degree classifies the homotopy classes of mappings from a sphere into itself. Bourgain, Brezis and Mironescu have obtained some linear estimates of the degree of a mapping by any critical first-order or fractional Sobolev energy. Similarly, maps from the three-dimensional sphere to the two-

dimensional spheres are classified by their Hopf invariant. Thanks to the Whitehead formula, Rivire has proved a sharp nonlinear control of the Hopf invariant by the first-order critical Sobolev energy. I will explain how a general compactness argument implies that sets that have bounded critical fractional Sobolev energy have bounded Hopf invariant and how we are obtaining in collaboration with Armin Schikorra (Pittsburgh) a sharp nonlinear estimates in critical fractional Sobolev spaces whose order is close to 1.

Nonlinear elliptic equations with measure valued absorption potential

Laurent Véron

University of Tours, France

We study the semilinear elliptic equation

$$-\Delta u + g(u)\sigma = \mu$$

with Dirichlet boundary condition in a smooth bounded domain where σ is a nonnegative Radon measure, μ a Radon measure and g is an absorbing nonlinearity. We show that the problem is well posed if we assume that σ belongs to some Morrey class. Under this condition we give a general existence result for any bounded measure provided g satisfies a subcritical integral assumption. We study also the supercritical case when $g(r) = |r|^{q-1}r$, with $q > 1$ and μ satisfies an absolute continuity condition expressed in terms of some capacities involving σ. Extensions to more general operators and nonlinear boundary value problems are sketched.