Abstracts

Plenary talks (50 minutes + questions)

Neumann domains
Ram Band (Technion)

The nodal set of a Laplacian eigenfunction forms a partition of the underlying manifold. An alternative partition, based on the gradient field of the eigenfunction, is via the so called Neumann domains. A Neumann domain of an eigenfunction is a connected component of the intersection between the stable manifold of a certain minimum and the unstable manifold of a certain maximum. We introduce this subject, discuss various properties of Neumann domains and point out the similarities and differences between nodal domains and Neumann domains. The talk is based on joint works with Sebastian Egger, David Fajman and Alexander Taylor.

Volume growth estimates on complete Riemannian manifolds
Gilles Carron (Nantes)

Polynomial bound for the volume growth of geodesic ball of complete Riemannian manifolds is a first coarse estimate that could lead to a better understanding of the geometry at infinity. In this talk, I will try to survey different technics : comparison geometry, spectral theory, functional analysis, harmonic analysis.

Nonlinear diffusions on negatively curved manifolds
Gabriele Grillo (Polytecnico di Milano)

We review some recent results on properties of solutions to classes on nonlinear diffusions on manifolds, such manifolds being mainly of Cartan-Hadamard type. Basic issues are existence, uniqueness (or nonuniqueness) of solutions, asymptotic behaviour, possible blow-up versus global existence properties. Some connection with nonlinear elliptic equations, as well as with properties of the linear heat equation on the same manifolds when uniqueness is concerned, are singled out. The results have been obtained in collaboration with M. Muratori, F. Punzo, J.L. Vazquez.

Some progress on Riesz transform and geometric PDEs
Renjin Jiang (Tianjin)

In this talk, I shall report some recent results regarding the boundedness of Riesz transform, regularity of harmonic functions and heat kernels on manifolds as well as metric measure spaces. It is well known that Riesz transform is intimately connected to heat kernels. It turns out the regularity of harmonic functions also plays a key role in the study of Riesz transform. The talk is based on some recent result joined with T. Coulhon, P. Koskela, F. Lin and A. Sikora.

Discrete spectrum for graphs
Matthias Keller (Potsdam)

Roughly speaking, purely discrete spectrum for the Laplacian on Riemannian manifolds is known to occur for two reasons : Either the manifold is "very bounded" or "very unbounded". This can be made precise by asking for (pre-)compactness or uniform unbounded curvature growth of the manifold. For graphs very similar phenomena can be studied. We review two classes of infinite graphs whose Laplacian has purely discrete spectrum. One class can be understood to be pre-compact in an appropriate sense and the other one to admit some kind of unbounded curvature growth. (This is based on various works together with Michel Bonnefont, Sylvain Golenia, Daniel Lenz, Marcel Schmidt, Melchior Wirth and Radoslaw Wojciechowski).

Ricci flow on singular spaces
Klaus Kröncke (Hamburg)

In this talk, I will report on joint work with Boris Verman on the Ricci flow on manifolds with isolated conical singularities. We construct a Ricci de Turck flow which starts sufficiently close to a Ricci-flat metric with isolated conical singularities and converges to a singular Ricci-flat metric under an assumption of integrability, linear and tangential stability. Additionally, we provide a characterization of conical singularities satisfying tangential stability and discuss examples where the integrability condition is satisfied. Finally, I will discuss properties of Perlman's entropies on manifolds with isolated conical singularities.

The systole of large genus minimal surfaces in positive Ricci curvature
Henrik Matthiesen (Chicago)

We prove that the systole (or more generally, any k-th homology systole) of a minimal surface in an ambient three manifold of positive Ricci curvature tends to zero as the genus of the minimal surfaces becomes unbounded. This is joint work with Anna Siffert.

A "next eigenvalue" estimate for the Hilbert-Brunn-Minkowski operator
Emanuel Milman (Technion)

In 1910, Hilbert applied his newly emergent spectral theory of integral operators to derive a proof of the Brunn-Minkowski inequality. It turns out that the Brunn-Minkowski inequality is precisely equivalent to an explicit spectral-gap estimate for a certain second-order elliptic operator L which may be associated with any (smooth) convex body in $\mathbb{R}^n$.

Many years later, it was proposed by Firey to replace the Minkowski addition of convex sets by its L^p counterpart, in which the support functions are added in L^p-norm, and this theory was developed for $p \geq 1$ by Lutwak in the 1990's. Recently, Böröczky, Lutwak, Yang and Zhang have proposed to extend this theory further to encompass the range $p \in [0,1)$. In particular, they conjectured an L^p-Brunn–Minkowski inequality for *origin-symmetric* convex bodies in that range, which constitutes a strengthening of the classical Brunn-Minkowski inequality.

It turns out that this conjecture is equivalent to establishing a "next eigenvalue" gap estimate for L (for even test functions), which should hold uniformly for all (origin-symmetric) convex bodies in $\mathbb{R}^n$. Our main result confirms the existence of such a uniform spectral-gap, thereby verifying the BLYZ conjecture (locally) for all origin-symmetric convex bodies in $\mathbb{R}^n$ and $p \in [1 - \frac{c}{n^{3/2}}, 1)$.

Based on joint work with Alexander Kolesnikov (Moscow).

New geometric examples of spaces satisfying a synthetic curvature bound
Ilaria Mondello (Créteil)

In the recent years, many striking results have been proven in the setting of metric measure spaces satisfying a curvature-dimension condition in the sense of Lott-Sturm-Villani, called RCD spaces, which share a multitude of geometric and analytic properties with Riemannian manifolds carrying a lower Ricci bound. In this talk we are interested in spaces that are in-between the smooth world of Riemannian manifolds and the very general setting of RCD spaces, that is singular manifolds, not necessarily arising as GH-limits. We will consider stratified spaces, which generalize manifolds with isolated conical singularities, and show a a geometric criterion for them to satisfy the RCD(K,N) condition or being Alexandrov spaces. This is a joint work with J. Bertrand (U. Paul Sabatier, Toulouse), C. Ketterer (U. of Toronto) and T. Richard (U. Paris Est Créteil).

Bernstein type inequalities via the heat semigroup
El Maati Ouhabaz (Bordeaux)

We prove general Bernstein type inequalities for gradient of eigenfunctions on Riemannian manifolds or Euclidean domains. The method relies on the heat semigroup and estimates for the corresponding heat kernel. Talk based on a joint work with R. Imekraz (U. Bordeaux).

Spectral asymptotics for the Steklov problem on curvilinear polygons
Iosif Polterovich (Montréal)

We will discuss the asymptotic behavior of Steklov eigenvalues and eigenfunctions on planar domains with corners. This problem is closely linked to some old questions in hydrodynamics, as well as to the study of quantum graphs. Interestingly enough, the arithmetic properties of the angles at the corner points have a significant effect on spectral asymptotics. The talk is based on a joint work in progress with Michael Levitin, Leonid Parnovski and David Sher.

Existence and regularity of an optimal shape for the Laplacian with a drift
Emmanuel Russ (Grenoble)

Let $D \subset \mathbb{R}^d$ be a fixed domain, $m \in (0, |D|)$ and $\tau \geq 0$. For all quasi-open sets $\Omega \subset D$ and all vector fields $V \in L^\infty(\Omega, \mathbb{R}^d)$, let $\lambda_1(\Omega, V)$ be the principal eigenvalue of the operator $L = -\Delta + V \cdot \nabla$ in Ω under Dirichlet boundary condition. We prove that the following minimization problem :

$$\min \left\{ \lambda_1(\Omega, V) \ : \ \Omega \subset D \text{ quasi-open, } |\Omega| \leq m, \ \|V\|_{L^\infty} \leq \tau \right\}$$

has a solution. If V is furthermore assumed to be the gradient of a Lipschitz function, we describe the regularity of optimal domains. This is a joint work with B. Trey and B. Velichkov (U. Grenoble Alpes).

Embedding metric measure spaces by the heat kernel
David Tewodrose (Cergy-Pontoise)

In this talk, I will present the main results of a recent article written in collaboration with L. Ambrosio (SNS Pise), S. Honda (Tohoku University) and J. Portegies (Eindhoven University) in which we adapt the following theorem of P. Bérard, G. Besson and S. Gallot to the context of metric measure spaces : for any closed (i.e. compact without boundary) Riemannian manifold, there exists a family of smooth embeddings $(\Phi_t)_{t>0}$ of M into a Hilbert space whose corresponding pull-back metrics $(g_t)_{t>0}$ converge to g with a first-order term involving the Ricci and scalar curvatures of (M, g). I will explain how this result extends to the class of compact $RCD(K, N)$ spaces and present a few perspectives.

Short talks (25 minutes + questions)

On the statistics of counting nodal points and Neumann points for quantum graphs

Lior Alon (Technion)

In this talk I will briefly go over the definitions and results of previous works on the nodal count statistics for quantum graphs. I will then introduce a similar concept of counting extremal points, called the Neumann points, and the regions they defined which are called Neumann domains. I will discuss some spectral and geometric properties of Neumann domains, and then I will state our results regarding the statistics of the Neumann count of eigenfunctions and its symmetry. I will finish by a nice inverse problem result showing topological and combinatorial properties of the underlying graph that can be extracted from the Neumann and nodal counts.

This talk is based on a joint work with R. Band (Technion) and G. Berkolaiko (Texas A&M).

Bound states of a pair of particles on the half-line with a general interaction potential

Sebastian Egger (Technion)

We study an interacting two-particle system on the positive half-line. We focus on spectral properties of the Hamiltonian for a large class of two-particle potentials. We characterize the essential spectrum and prove, as the main result, the existence of eigenvalues below the bottom of it. We also prove that the discrete spectrum contains only finitely many eigenvalues.

On Graph Limits and the Essential Spectrum of Schrödinger Operators

Latif Eliaz (Technion)

It is known that the essential spectrum of a Schrödinger operator H on $\ell^2(\mathbb{N})$ is equal to the union of the spectra of right limits of H. The natural generalization of this relation to $\mathbb{Z}^n$ is known to hold as well. In this talk we study the possibility of generalizing this characterization of $\sigma_{ess}(H)$ to graphs. The natural generalization of the concept of right limits to graphs turns out to be a Schrödinger operator version of Benjamini-Schramm limit. We show that the general characterization of the essential spectrum fails, while presenting natural families of models where it still holds.

A Riesz Decomposition Theorem for Schrödinger Operators on Graphs

Florian Fischer (Potsdam)

In the classical potential theory on the Euclidean space and in the potential theory of transient Markov chains a unique decomposition of superharmonic functions into a harmonic and a potential part is well-known. In this talk the basic concepts and ideas to gain such a decomposition for Schrödinger operators on graphs will be shown. This is a joint work with Matthias Keller.

Spectral flow for elliptic boundary value problems on compact surfaces

Marina Prokhorova (Technion)

A one-parameter family of self-adjoint Fredholm operators has a well-known integer-valued invariant, the spectral flow. It counts with signs the number of operators' eigenvalues passing through zero with the change of parameter. For loops of elliptic operators on a closed manifold, the spectral flow was computed by Atiyah, Patodi, and Singer (1976) in terms of topological data of a loop. But if a manifold has non-empty boundary, then boundary conditions come into play, and situation becomes more complicated.

In the talk I will explain how to compute the spectral flow for loops (or, more generally, paths with ends conjugated by a unitary automorphism) of first-order self-adjoint elliptic operators with classical boundary conditions in two-dimensional case, that is when a manifold is a compact surface. The answer is given in terms of the topological data over the boundary. The talk is based on my preprint arXiv :1703.06105

Sobolev-Type Inequalities and Eigenvalue Growth on Graphs with Finite Measure

Melchior Wirth (Jena)

In this talk, lower bounds on the eigenvalue growth of infinite graphs with discrete spectrum are presented. Under the assumption that the corresponding Dirichlet forms satisfy certain Sobolev-type inequalities and that the total measure is finite, a Weyl-type lower bound for the eigenvalue growth is given. In this sense, the graph Laplacians display similarities to elliptic operators on bounded domains. It is shown by examples that corresponding upper bounds cannot be established. (This talk is based on joint work with Bobo Hua, Matthias Keller, and Michael Schwarz).