LINEAR FLOWS ON THE SPACE OF LATTICES

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A lattice in the plane is the $\mathbb{Z}$-span of two linearly independent vectors in the plane. The group, $\mathrm{GL}(2,\mathbb{R})$, of linear transformations on the plane acts transitively on the space of lattices in the plane which gives this space a natural topology.

In this project we will study the space of lattices in the plane as a dynamical system. We will try to classify orbits of (one-parameter unipotent and diagonalizable) subgroups of the group $\mathrm{GL}(2,\mathbb{R})$ focusing on topological properties of such orbits (such as closed, compact and dense orbits). We will proceed the problem by emphasizing the fundamental differences of such properties between the orbits of the unipotent and diagonalizable subgroups. If time permits we will also study analogous cases in positive characteristics.

Prerequisites. We will not assume anything but undergraduate level in linear algebra, group theory and general topology. However, since the week is shorter than it seems, some preparation will be beneficial for all of us. In particular, knowing the Jordan form of a matrix, the concept of a quotient topology and Baire's category theorem will be useful. Familiarity with basic geometry of numbers will be helpful (e.g. the topology on the space of lattices in $\mathbb{R}^n$ and Mahler's compactness criterion). For these materials, we recommend Chapter I and V of the book **An introduction to the geometry of numbers** by Cassels. Some algebraic number theory (e.g. quadratic number fields, their ring of integers and Dirichlet's unit theorem) and complex analysis (e.g. Möbius maps on the Riemann sphere) might be useful for the later parts of this project.