How to solve a periodic tangram?

Abstract: The tangram is a dissection puzzle consisting of several flat shapes, called tans, which are put together to form a given shape in the plane (say, a square), without rotations.

You can experience this in the application store on your smartphone: https://play.google.com/store/apps/details?id=com.littlebeargames.tangram

In a periodic tangram the shape is on the flat torus, that is, periodic in both directions.

More generally, the shape can be a function $T$ on $\mathbb{R}^2$ which is $2\pi \times 2\pi$ periodic, namely $T(x, y) = T(x + 2k\pi, y + 2n\pi)$ for any $x, y \in \mathbb{R}^2$, $k, n \in \mathbb{Z}$. For example, $T$ can be the constant $T \equiv 1$. The tans are periodic functions as well $T_i$, $i = 1, \ldots, N$ (for example, characteristic functions of domains in the $2\pi$ square). Each tan can be shifted by $z = (z_1, z_2) \in \mathbb{R}^2/\mathbb{Z}^2$, and $T_i^{(z)}(x, y) \equiv T_i(x + z_1, y + z_2)$. The solution of the tangram is a set of $N$ shifts $z_1, \ldots, z_N$ such that

$$T = \sum_{i=1}^{N} T_i^{(z_i)}.$$

The periodic structure enable us to call good old Fourier for help and convert the problem to an algebraic one, represented by a system of homogeneous polynomial equation. The project is to write down this system and find an efficient numerical method to solve it.