

Technion summer projects 2020

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Quantized multiplicities for representations of p -adic groups

Smooth representations of the groups $GL_n(F)$, where F is a p -field, are a rich source of structure and symmetry, that appear in number theory on one hand and in quantum algebras on the other.

Taking finite-length representation of all groups $\{GL_n(F)\}_n$ gives a Grothendieck ring which can be studied algebraically. The structure of this ring and the position of the irreducible representations inside it, mimic the quantum group $U_q(\mathfrak{sl}_N)$ and the canonical basis inside it.

The latter ring has the additional quantized structure, that is, multiplication of elements by the formal variable q . Pulling these shifts back to representation theory, we see a hidden structure of (\mathbb{Z}) -graded representations. This can be explained by passing through some non-trivial categorical equivalences.

In this project, I suggest exploring those gradings from a computational point of view on interesting classes of representations, that are defined by combinatorial data. These efforts should, as a first step, produce formulas for graded degrees, using Kazhdan-Lusztig polynomials and quantum shuffle algebra computations. Specifically, I would like to explore the link between degrees of irreducible subquotients and the lattice of sub-representations.

Related to this, we can also explore computational links between q -characters of representations of quantum affine algebras and characters of representations of affine Hecke algebras.

While those concepts rely on non-negligible amount of theoretic background, the project is intended to give a feeling of a wide range of themes in representation theory for a student without specialized expertise.

Participating students should be familiar with ring theory, and preferably, be comfortable with modules over associative algebras. Ability to use mathematical computational software would be useful.

Some random related literature: [1, 3, 2].

References

- [1] J. Brundan and A. Kleshchev. Blocks of cyclotomic Hecke algebras and Khovanov-Lauda algebras. *Invent. Math.*, 178(3):451–484, 2009.
- [2] M. Gurevich. An identity of parabolic Kazhdan-Lusztig polynomials arising from square-irreducible modules. *Journal of the Australian Mathematical Society* <https://doi.org/10.1017/S144678871900017X>, 2019.
- [3] B. Leclerc, M. Nazarov, and J.-Y. Thibon. Induced representations of affine Hecke algebras and canonical bases of quantum groups. In *Studies in memory of Issai Schur (Chevaleret/Rehovot, 2000)*, volume 210 of *Progr. Math.*, pages 115–153. Birkhäuser Boston, Boston, MA, 2003.