

# How hot can a JPEG image be?

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How can we take an digital image and reduce its size while keeping its quality and neatness? While there have been several attempts made before the 1990s, two compressions methods were invented that are still with us today: the JPEG for pictures in 1992 and the MPEG-1 Audio-Layer III (also known as MP3) in 1993. These two methods were based on the properties of the human ears and eyes which had been discovered in the previous years, as well as a method invented in 1972: the Discrete Cosine Transform, or DCT.

In this project, we will explore the JPEG standard in more details. We will define the basic properties of the DCT as well as give a complete description of the JPEG algorithm. We will then explore what images are well-suited to be easily compressed, as well as the main form of compression errors for badly-behaved images.

We will also explore (depending on the knowledge and/or interest of the participants) the connexions between the JPEG standard and some deep ideas in various fields of mathematics. Namely, we will talk about the link between compression of pictures and the heat flow going through a square membrane, Weyl's law, and connexions to functional analysis and Hilbert spaces.

If we have the time, we will try to come up with ideas on how to improve JPEG, as well as some numerical play on Matlab.

Prerequisites: linear algebra. If participants have some knowledge of fourier series it will allow us to move faster but it is not mandatory as we will cover the basics in this project. Use of Matlab is a plus but is also not mandatory.

## References

Rousseau, Christiane & Saint-Aubin, Yvan, *Mathematics and Technology*. Springer Undergraduate Texts in Mathematics and Technology, Springer, 2008.