Summer Project 2021

Google navigation in Manhattan

You are located at the intersection of 0 street and 0 avenue in an infinite Manhattan network of streets/avenues and wish for some reason to drive your car one block to the next intersection (say, 0 street and avenue 1). Assume:

1. There is a flux of S cars per minute leaving the parking lot (0,0) and arriving to (0,1), and the resulting traffic is stationary.

2. The time it take to drive one block depends on the flux of cars trough this block: If the flux is s then the time is $\alpha + \beta s$ where $\alpha \geq 0, \beta > 0$ are prescribed (independent of the block).

3. You use a Google navigation problem which indicates the expected time of arrival along each route. Of course you (and everybody else) choose the fastest one. If there is more than one such route, you choose between them at random. In particular, all *active* routes take the same time.

4. There are no other parking lots in Manhattan. As a result, the incoming flux at each intersection equals the outgoing flux (Kirkhoff condition).

The question is: What is the time $T(S)$ for arrival, as a function of the total flux S? It may not be possible to get an explicit expression, but, perhaps you can try find an asymptotic time as $S \to \infty$, either by analytic consideration or use simulation to get some intuition. What about "higher dimensional Manhattan" or some general infinite graph?

An interesting fact can be useful: This problem is equivalent, in some sense, to a *quadratic, constraint optimization problem* resulting from a *social Google program* which aims at minimizing the *average* arrival time. This suggests an official numerical algorithm for calculation, so you should be ready to make some "dirty" numerical work...

Recommended reading for theoretical background: Chapter 18 in *Algorithmic Game Theory* by Noam Nisan (available on line at the library).