Distribution of big balls in infinite groups on finite quotients

and what geometry and probability has to do with it?

Mentor: Ilya Gekhtman

Let G be a finitely generated infinite group acting transitively on a finite graph. We are interested in how orbits of G are asymptotically distributed in X. There are two main ways to formulate this question.

1. Take a random walk on G, and look at the probability that after n steps you are at a given point p of X. Under mild conditions, it can be shown that this probability converges to $\frac{1}{|X|}$, in other words random walks on G equidistribute in X. This is a consequence of the so-called Perron-Frobenius theorem for positive matrices.
2. Pick a metric on G, for example a word metric coming from some finite symmetric generating set. Look at the proportion of elements in a ball of radius N around the identity whose images lie at a particular point p in X. In other words, how are big balls in G distributed in X? Does their distribution converge to some distribution in X? Under what condition does it converge to the uniform distribution on X?

In general this is a difficult, even intractable problem. We will attack it when G is a nonabelian free group , and X happens to be a group. A key tool we will try use is that free groups admit an automatic structure, which roughly means that geodesics in their word metrics are parametrized by paths in a finite directed graph.

We will use this structure to reduce the study of balls in the Cayley graph to studying a collection of associated random walks on the group. If time remains we will try to generalize these techniques to other groups admitting automatic structure including fundamental groups of closed surfaces and more generally a class called hyperbolic groups. The study of automatic structures for such groups is a central subject in modern group theory, geometry, topology, and dynamical systems.

Prerequisites: Basic group theory. Some knowledge of elementary probability theory is helpful but not essential.