Percolation of Rooted Spanning Forests in Nonamenable Graphs

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Random spanning trees and spanning forests are fundamental objects in probability and computer science. Here, a tree is an acyclic subgraph of a given fixed graph G and a forest is a subgraph which is a disjoint union of trees. Spanning means that all vertices are in the subgraph.

For a given finite graph G consider the probability measure on spanning forests in which a forest F is chosen with a weight $h^{C(F)} \prod_{T \in F} |T|$, where $C(F)$ is the number of components of F, $T \in F$ indicates T is a connected component of$F$ and $|T|$ is the number of vertices in T.

In this project we will study the typical size of a random rooted spanning forest on infinite graphs. It is known that on the Euclidean lattice Z^d, every tree in a random sample is finite almost surely, for any choice of h. However, there is good reason to believe that if the graph G is "as far from Euclidean as possible", e.g. a d-regular tree, one should find infinite trees in a random sample. The technical name for "as far from Euclidean as possible" is non-amenable. So we ask: Is it the case that on any bounded degree, non-amenable graph G, for the parameter h small enough a random sample T has infinite components?

To answer this question, students will need to understand a number of elementary and beautiful ideas: Kirchoff's MatrixTree Theorem for expressing rooted spanning forests as determinants of adjacency matrices, Wilson's Algorithm for simulating random spanning forests using Loop-Erased Random Walk, non-amenable graphs and how they typically come up and of course how to sample spanning forests on infinite graphs.