VARIABLE LEBESGUE SPACES VERSUS L_p SPACES WHEN IT COMES TO FIXED POINT THEORY

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Variable Lebesgue spaces $L^{p(\cdot)}(\Omega)$ can be considered as a generalization of classical Lebesgue spaces $L^p(\Omega)$ when the exponent p is allowed to be a variable measurable function $p(\cdot)$. Although variable Lebesgue spaces can be included within the larger family of Musielaz-Orlicz spaces, during the past three decades many researches have felt very much attracted to this variable extension of Lebesgue spaces. In particular, this field has experimented a burgeoning development due in part to the discovery of many applications of $L^{p(\cdot)}(\Omega)$ to PDEs connected with different problems of physical nature (see references [1, 2, 4]). In fact, properties of variable Lebesgue spaces related to some areas such as harmonic analysis or partial differential equations have been widely studied in [1, 2, 4]. It is important to emphasize that variable Lebesgue spaces share many standard properties from classical Lebesgue spaces but, at the same time, differ from them in several subtle and interesting ways.

A fixed point theory focusing on these particular function spaces seemed to be at an early stage. In this talk we will introduce variable Lebesgue spaces and we will analyze the existence of fixed points for nonexpansive mappings in the variable context $L^{p(\cdot)}(\Omega)$. The behaviour of the exponent function $p(\cdot)$ becomes essential in determining the fulfilment of the (weakly) fixed point property. We will check that, although our results may resemble those already known for the standard Lebesgue L^p spaces, we find out some pleasing and unexpected surprises that, once more, allow us to raise new open problems within the scope of metric fixed point theory. Most of the content of this talk can be found in [3].

References

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[3] T. Domínguez Benavides, M.A. Japón, *Fixed point properties and reflexivity in variable Lebesgue spaces.* J. Funct. Anal. 280 (2021), no. 6, Paper No. 108896, 22 pp.

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