The history of PQR

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Abstract:

For various mathematical reasons, we are interested in projections, in particular in orthogonal projections in Hilbert spaces.
If we consider a single orthogonal projection $P$ of a Hilbert space onto a closed vector subspace $F$, $P^n(x)$ is always the same point (for $n \geq 1$) because $P^2 = P$. Now if we consider a second projection $Q$ on the closed subspace $G$, then the sequence of iterated projections which will be of the form $(P \circ Q)^n(x)$ or $(P \circ Q)^2(x)$ converges strongly to a point of intersection of $F$ and $G$. Let's go to 3 projections $P$, $Q$ and $R$. What happens when we look at the iterates? If we take a periodic sequence in $P$, $Q$ and $R$, then the sequence also converges strongly. But what would happen if we take a random sequence in $P$, $Q$ and $R$? Would there be also a strong convergence? Ando showed that such a sequence converges weakly. Ron Bruck made a conjecture that if true would show strong convergence. We will see the surprising solution of this problem.