<u>**The history of PQR**</u>

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Abstract:

For various mathematical reasons, we are interested in projections, in particular in orthogonal projections in Hilbert spaces.
If we consider a single orthogonal projection P of a Hilbert space onto a closed vector subspace F, $P^n(x)$ is always the same point (for $n >= 1$) because $P^2=P$. Now if we consider a second projection Q on the closed subspace G, then the sequence of iterated projections which will be of the form $(P\ Q)^n\ (x)$ or $(P\ Q)^P\ (x)$ converges strongly to a point of intersection of F and G. Let's go to 3 projections P, Q and R. What happens when we look at the iterates? If we take a periodic sequence in P, Q and R, then the sequence also converges strongly. But what would happen if we take a random sequence in P, Q and R? Would there be also a strong convergence? Ando showed that such a sequence converges weakly. Ron Bruck made a conjecture that if true would show strong convergence. We will see the surprising solution of this problem.