POLYNOMIAL ESTIMATES FOR THE METHOD OF CYCLIC PROJECTIONS IN HILBERT SPACES

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We study the method of cyclic projections when applied to closed and linear subspaces M_i, $i = 1, \ldots, m$, of a real Hilbert space $\mathcal{H}$ under the assumption that $\sum_{i=1}^{m} M_i^{\perp}$ is not closed. We show that the proximity function involving the average distance to individual sets enjoys a polynomial behaviour $o(k^{-1/2})$ along the trajectory of the generated iterates. Moreover, when the starting points are chosen from the subspace $\sum_{i=1}^{m} M_i^{\perp}$, the above-mentioned proximity function satisfies an improved upper bound $\mathcal{O}(k^{-1})$. Surprisingly, in the latter case, our result yields a polynomial rate of convergence $\mathcal{O}(k^{-1/2})$ for the method of cyclic projections itself. Moreover, all of the aforementioned rates are best possible in the sense that the power of the corresponding polynomials cannot be improved (decreased by any positive epsilon).

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Key words and phrases: Product space; Rates of asymptotic regularity; Rates of convergence.