

# Noncommutative Analysis at the Technion

In honor of Paul Muhly for his research contributions and his  
leadership

Technion, Haifa, Israel

June 26 – July 1, 2022

# Useful Information

- On Sunday evening there will be a reception at the Dan Panorama Hotel. The reception will be a buffet style dinner, food will be available 19:00-21:00.
- On Wednesday evening there will be a banquet in which we will gather to honor Paul Muhly and celebrate his contributions and leadership. The banquet will take place at 18:00 in the lounge on the 8th floor of the Amado building.
- All the lectures will take place in Room 232 on the second floor of the Math Building (Amado).
- Most talks are scheduled for 50 minutes, which means 45 minutes plus 5 minutes for questions. The colloquium talk on Monday is 50+10.
- The Faculty of Mathematics is located in the Amado Building. The Amado building is connected to the Segoe Building, which is the home of Architecture and Town Planning.
- Some of the guests will be staying at the Dan Panorama Hotel, which is in a nice part of town, and some will be staying at the Forchheimer guesthouse, which is located on campus. If you are staying at Dan Panorama, a shuttle will take to the Technion for the talks and back every day. If you are staying at Forchheimer, a taxi will take you to the hotel for the reception and back on Sunday evening.
- Information on how to get to the hotel/guesthouse using public transportation can be found in this link.
- The Forchheimer guesthouse is located in square D3 of this map.
- Lunch and Coffee breaks will be served in the Faculty Lounge on the 8th floor of the Amado Building. There is a cafeteria on the third floor of the Ullman building.
- You can also hang out in the faculty lounge on the 8th floor of Amado, or in the library, which is on the fourth floor. This is also a kitchenette and sitting area on the fifth floor.
- **Covid-19:** We will follow Ministry of Health guidelines regarding Covid-19. See here for regulations regarding entry to Israel. According to the current regulations, if you get infected then you need to go into isolation according to the instructions found here.

# Schedule

	<b>Sunday</b>
18:30	Pickup at Forchheimer (to the hotel)
19:00 – 21:00	Reception at the Dan Panorama Hotel
21:15	Pickup back to the Forchheimer

	<b>Monday</b>
8:15	Pickup the hotel
9:00 – 9:10	Opening
9:10 - 10:00	Guy Salomon <i>Stability of Cuntz–Krieger relations</i>
10:00 – 10:30	Coffee break
10:30 - 11:20	Chaim Schochet <i>Tilings, <math>K</math>-theory, and Cohomology</i>
11:20 – 11:30	Short break
11:30 - 12:20	Ami Viselter <i>Lévy processes on quantum groups and examples</i>
12:20 – 14:10	Lunch break
14:10 - 15:00	Jaydeb Sarkar <i>Quotient modules of the Hardy module on the polydisc</i>
15:00 – 15:30	Coffee break
15:30 - 16:30	Michael Hartz <i>von Neumann’s inequality on the disc and on the ball</i>
16:30 – 16:40	Short break
16:40 - 17:30	Raphael Clouatre <i>Non-commutative peak points in operator algebras</i>
17:45	Pickup back to the hotel

<b>Tuesday</b>	
8:15	Pickup the hotel
9:10 - 10:00	Ilan Hirshberg <i>Toward a dimension theory for amenable groupoids and their <math>C^*</math>-algebras</i>
10:00 – 10:30	Coffee break
10:30 - 11:20	N. Christopher Phillips <i>An invitation to mean dimension of a dynamical system and the radius of comparison of the crossed product</i>
11:20 – 11:30	Short break
11:30 - 12:20	Shirly Geffen <i>Pure <math>C^*</math>-algebras of stable rank one</i>
12:20 – 14:20	Lunch break
14:20 - 15:10	Mariusz Tobolski <i>Noncommutative numerable principal bundles from group actions on <math>C^*</math>-algebras</i>
15:10 – 15:40	Coffee break
15:40 - 16:30	Marcelo Laca <i>Universal Toeplitz algebras and their boundary quotients</i>
16:30 – 16:40	Short break
16:40 - 17:30	Camila Sehnem <i><math>C^*</math>-envelopes of tensor algebras of product systems</i>
17:45	Pickup back to the hotel

	<b>Wednesday</b>
8:15	Pickup the hotel
9:10 - 10:00	Rachael M. Norton <i>Cartan subalgebras of twisted groupoid <math>C^*</math>-algebras including higher rank graph <math>C^*</math>-algebras</i>
10:00 - 10:30	Coffee break
10:30 - 11:20	Bartosz Kwaśniewski <i>Aperiodicity for Hilbert bimodules and <math>C^*</math>-inclusions</i>
11:20 - 11:30	Short break
11:30 - 12:20	Valentin Deaconu <i>Self-similar groupoid actions and <math>C^*</math>-algebras</i>
12:20 - 14:20	Lunch break
14:20 - 15:10	Marius Ionescu <i>Groupoids, Unitary Extensions and Wavelets</i>
15:10 - 15:40	Coffee break
15:40 - 16:30	Chris Bruce <i><math>C^*</math>-algebras from algebraic semigroup actions</i>
16:30 - 16:40	Short break
16:40 - 17:30	Paul S. Muhly <i>Tensor Algebras as Algebras of Functions</i>
18:00 - 20:00	Banquet
20:15	Pickup back to the hotel

<b>Thursday</b>	
8:15	Pickup the hotel
9:10 - 10:00	Michael Jury <i>An optimal approximation problem for noncommutative polynomials</i>
10:00 – 10:30	Coffee break
10:30 - 11:20	Eli Shamovich <i>Noncommutative rational functions on the unit ball</i>
11:20 – 11:30	Short break
11:30 - 12:20	Marina Prokhorova <i>Unbounded Fredholm operators</i>
12:20 – 14:20	Lunch break
14:20 - 15:10	Elias Katsoulis <i>Isomorphisms and stable isomorphisms of non-selfadjoint operator algebras</i>
15:10 – 15:40	Coffee break
15:40 - 16:30	Daniel Markiewicz <i><math>E_0</math>-semigroups and boundary weight maps, the infinite dimensional case</i>
16:30 – 16:40	Short break
16:40 - 17:30	Jean Renault <i>Abelian twisted groupoid extensions</i>
17:30–17:45	Closing remarks
18:00	Pickup back to the hotel

# Abstracts

## **C\*-algebras from algebraic semigroup actions**

Chris Bruce

Each algebraic semigroup action naturally gives rise to a concrete C\*-algebra acting on a Hilbert space. I will explain a method for finding groupoid models for such C\*-algebras, and then present results characterizing certain properties of these groupoids in terms of the initial algebraic semigroup action. As an application, we obtain structural results for the C\*-algebras arising from several large classes of algebraic semigroup actions. This is joint work with Xin Li.

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## **Non-commutative peak points in operator algebras**

Raphael Clouatre

Classically, the Choquet boundary of a function algebra consists of points admitting a unique representing measure, and an old result of Bishop states that this coincides with the set of all peak points. One of the cornerstones of the theory of non self-adjoint operator algebras is Arveson's non-commutative analogue of the Choquet boundary, constructed via representations admitting a unique "representing" completely positive map. The corresponding analogue of peak points is comparatively much less developed.

In this talk, I will discuss some recent developments surrounding non-commutative peak points. I will also illustrate some applications of these ideas to various topics in operator algebras, such as the hyperrigidity conjecture, non-commutative interpolation and zero sets, and residual finite-dimensionality.

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## Self-similar groupoid actions and $C^*$ -algebras

Valentin Deaconu

Given a self-similar groupoid action  $(G, E)$  on the path space of a finite graph, we prove some properties of the associated Exel-Pardo étale groupoid  $\mathcal{G}(G, E)$  and its  $C^*$ -algebra  $C^*(G, E)$ . We review some facts about groupoid actions, skew products and semi-direct products, and describe a general strategy to compute the  $K$ -theory of  $C^*(G, E)$  and the homology of  $\mathcal{G}(G, E)$  in certain cases.

We also study the analogue of the Higman-Thompson group associated to  $(G, E)$  using  $G$ -tables and relate it to the topological full group of  $\mathcal{G}(G, E)$ , which is isomorphic to a subgroup of unitaries in the algebra  $C^*(G, E)$ .

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## Pure $C^*$ -algebras of stable rank one

Shirly Geffen

The Cuntz semigroup has become increasingly important in connection with Elliott's classification program. Our work further contributes to the evidence that a  $C^*$ -algebraic dimension type property, so-called stable rank one, is deeply connected to purely algebraic properties of the Cuntz semigroup: almost divisibility and almost unperforation. In particular, we can show that, under natural assumptions, stable rank one can be recovered from almost divisibility and almost unperforation.

This is joint work in progress with Wilhelm Winter.

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## von Neumann's inequality on the disc and on the ball

Michael Hartz

von Neumann's inequality provides a fundamental link between analytic functions on the unit disc and contraction matrices, or more generally contraction operators on Hilbert space. It asserts that if  $T$  is a contraction and  $p$  is a polynomial, then

$$\|p(T)\| \leq \sup\{|p(z)| : |z| \leq 1\}.$$

The multivariable setting turns out to be significantly more complicated. In particular, it is known that the naive version of von Neumann's inequality on the Euclidean unit ball is false.

I will talk about the original inequality and some of the challenges in several variables. Moreover, I will mention a multivariable inequality for matrices, with constants depending on the size of the matrix. Based on joint work with Stefan Richter and Orr Shalit.

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# Toward a dimension theory for amenable groupoids and their $C^*$ -algebras

Ilan Hirshberg

The notion of nuclear dimension for  $C^*$ -algebras, introduced by Winter and Zacharias, is a key concept in structure and classification theory for  $C^*$ -algebras. There is therefore considerable interest in finding conditions on groupoids of interest (particularly transformation groupoids) which ensure that the associated  $C^*$ -algebra has finite nuclear dimension. I will give a survey of some known results, focusing on joint work with Jianchao Wu (both in progress and published). The talk will be intended for researchers who might not work in structure and classification theory, and in particular I will not assume that the audience knows about nuclear dimension.

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## Groupoids, Unitary Extensions and Wavelets

Marius Ionescu

There has been a rich interplay between representations of Cuntz relations and wavelets, going back to work by Bratelli and Jorgensen, and, subsequently, work by Jorgensen, Dutkay, and Pedersen. More recently, Larsen and Raeburn, Packer and Rieffel, and Packer with a host of collaborators have pursued a rich program in which they studied connections between operator algebras and (generalized) wavelet analysis. Many of these studies rely on the understanding of  $C^*$ -algebras attached to local homeomorphisms and transfer operators. Some of the above mentioned constructions use special representations of Cuntz relations together with unitary extensions of specific isometries. In this project that is joint with Paul S. Muhly and that builds on our previous work, we use Deaconu-Renault groupoids and their so called blow ups together with  $C^*$ -correspondences attached to the local homeomorphism to analyze which representations are intrinsic to the given data. We show that most of the analysis that leads to wavelets depends only on a choice of a transfer operator for the local homeomorphism. An orthogonal family of transfer operators defines Cuntz isometries. Using inductive limit of  $C^*$ -correspondences, we prove that the minimal unitary extension lives in the multiplier algebra of what we call the infinite blow up of the original Deaconu-Renault groupoid. The unitary and the inductive system defines a natural projective multi-resolution analyses on the  $C^*$ -correspondence defined by the blow up. Using the general theory of inducing groupoid representations, we prove that many of the situations studied before in the literature can be recovered from our construction. We are currently working on applying our results to wavelet-like constructions on some spaces built out of fractals, called fractafolds.

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# An optimal approximation problem for noncommutative polynomials

Michael Jury

Motivated by recent work on optimal approximation by polynomials in the unit disk, we consider the following noncommutative approximation problem: for a polynomial  $f$  in  $d$  noncommuting arguments, find an nc polynomial  $p_n$ , of degree at most  $n$ , to minimize

$$c_n := \|p_n f - 1\|^2.$$

(Here the norm is the  $\ell^2$  norm on coefficients.) We show that  $c_n \rightarrow 0$  if and only if  $f$  is nonsingular in a certain nc domain (the *row ball*), and prove quantitative bounds. As an application, we obtain a new proof of the characterization of polynomials cyclic for the  $d$ -shift.

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## Isomorphisms and stable isomorphisms of non-selfadjoint operator algebras

Elias G. Katsoulis

In this talk we address isomorphisms and stable isomorphisms of various classes of operator algebras. We state and resolve the isomorphism problem for tensor algebras of unital multivariable dynamical systems. Specifically we show that unitary equivalence after a conjugation for multi-variable dynamical systems is a complete invariant for complete isometric isomorphisms between their tensor algebras. In particular, this settles a conjecture of Davidson and Kakariadis relating to work of Arveson from the sixties, and extends related work of Kakariadis and Katsoulis. We also address stable isomorphism of operator algebras, in connection with a recent work of Dor-On, Eilers and Geffen. Among others we show that if  $\mathcal{A}$  and  $\mathcal{B}$  are operator algebras with diagonals isomorphic to  $c_0$  and  $\mathcal{K}$  are the compact operators, then  $\mathcal{A} \otimes \mathcal{K}$  and  $\mathcal{B} \otimes \mathcal{K}$  are isometrically isomorphic if and only if  $\mathcal{A}$  and  $\mathcal{B}$  are isometrically isomorphic. If the algebras  $\mathcal{A}$  and  $\mathcal{B}$  satisfy an extra analyticity condition, a similar result holds with  $\mathcal{K}$  being replaced by any operator algebra containing the compact operators. Time permitting we will discuss other classes of operator algebras and their stable isomorphisms, including tensor algebras of multivariable dynamical systems. The above results come from various projects with C. Ramsey, E. Kakariadis and X. Lin.

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# Aperiodicity for Hilbert bimodules and $C^*$ -inclusions

Bartosz Kwasniewski

In their 2000 paper ‘On the Morita equivalence of tensor algebras’ Paul Muhly and Baruch Solel, inspired by the work of Kishimoto and Olesen-Pedersen, introduced a notion of aperiodicity for  $C^*$ -correspondences. I used this notion with Wojciech Szymanski to introduce aperiodic Fell bundles over discrete groups, and later together with Ralf Meyer we started a systematic study of aperiodic Hilbert bimodules and aperiodic  $C^*$ -inclusions. In a series of papers we clarified relationships between aperiodicity and topological freeness, pure outerness, detection of ideals, the almost extension property, uniqueness of pseudo-expectations and other conditions.

Aperiodicity turns out to be crucial for the study of ideal structure and pure infiniteness of  $C^*$ -algebras modeled by various crossed products. It can also be used to characterize noncommutative Cartan inclusions, regular  $C^*$ -irreducible inclusions and led to the introduction of the essential groupoid  $C^*$ -algebra for non-Hausdorff groupoids.

In my talk I will report on these results, that all grew up out of this condition introduced in 2000.

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## Universal Toeplitz algebras and their boundary quotients

Marcelo Laca

Let  $P$  be a submonoid of a group. We define a universal Toeplitz  $C^*$ -algebra  $\mathcal{T}_u(P)$ , given in terms of generators and relations, which coincides with Li’s semigroup  $C^*$ -algebra when  $P$  satisfies the independence condition and “behaves the right way” also when independence fails. Hence  $\mathcal{T}_u(P)$  can be used in general to study the reduced Toeplitz  $C^*$ -algebra  $\mathcal{T}_\lambda(P)$ . Along these lines, we obtain faithfulness and uniqueness theorems that generalize classical results of Coburn, Douglas, and Cuntz. We then focus on a universal boundary quotient  $\partial\mathcal{T}_u(P)$ , initially defining it via extra relations distilled from Sehnem’s covariance algebra, and give conditions on  $P$  for the reduced boundary quotient  $\partial\mathcal{T}_\lambda(P)$  to be purely infinite simple. As applications, we analyze non-maximal orders in number fields and right LCM monoids with nontrivial units. This is joint work with Camila F. Sehnem.

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# $E_0$ -semigroups and boundary weight maps, the infinite dimensional case

Daniel Markiewicz

An  $E_0$ -semigroup of  $B(H)$  is a particular kind of quantum Markov semigroup: it is a one parameter strongly continuous semigroup of unital  $*$ -endomorphisms of  $B(H)$ . This talk concerns the classification of  $E_0$ -semigroups up to cocycle conjugacy.

Robert T. Powers showed that every  $E_0$ -semigroup that possesses a strongly continuous intertwining semigroup of isometries arises (up to cocycle conjugacy) from a boundary weight map over  $K$  separable Hilbert space – in a certain sense, boundary weight maps are an adaptation of the concept of resolvents from classical semigroup theory to this context.

The case of  $q$ -pure boundary weight maps over  $K$  finite dimensional was classified completely in the paper: C. Jankowski, D. Markiewicz and R.T. Powers, “Classification of  $q$ -pure  $q$ -weight maps over finite dimensional Hilbert spaces”, *J. Funct. Anal.* 277 (2019), no. 6, pp. 1763–1867.

In this talk we present some recent developments in joint work with C. Jankowski and R. T. Powers, showing that the situation for infinite dimensional  $K$  presents some new phenomena.

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## Tensor Algebras as Algebras of Functions

Paul S. Muhly

In this talk, which is based on joint work with Baruch Solel, I describe how to realize a tensor algebra of a  $C^*$ -correspondence  $E$  over a  $C^*$ -algebra  $A$ ,  $T_A(E)$ , as a space of functions defined on  $\coprod_{\pi \in \widehat{A}} E^\pi$  with values in  $\coprod_{\pi \in \widehat{A}} B(H_\pi)$ , where  $\widehat{A}$  denotes the space of  $C^*$ -representations of  $A$ ,  $E^\pi$  is the so-called  $\pi$ -dual of  $E$ , and  $H_\pi$  is the Hilbert space of  $\pi$ . The function that an element  $f \in T_A(E)$  determines is fibred over  $\widehat{A}$  and is denoted  $\widehat{f} = \{\widehat{f}_\pi\}_{\pi \in \widehat{A}}$ . The function  $\widehat{f}_\pi$ , in turn, is a  $B(H_\pi)$ -valued, *polynomial function* defined on  $E^\pi$ . The relation between  $f$  and  $\widehat{f}$  is a straightforward generalization of the relation one learns about in school between polynomials with coefficients in a field and polynomial functions defined on affine spaces over the field. With these polynomial functions in hand, one can develop noncommutative analogues of analytic functions and function spaces. To the extent time permits, I will describe the state of these developments as we currently understand them.

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# Cartan subalgebras of twisted groupoid $C^*$ -algebras including higher rank graph $C^*$ -algebras

Rachael M. Norton

In this talk we identify Cartan subalgebras of reduced groupoid  $C^*$ -algebras whose multiplication is twisted by a circle-valued 2-cocycle. Specifically, we identify sufficient conditions on a subgroupoid  $S \subset G$  under which the twisted  $C^*$ -algebra of  $S$  is a Cartan subalgebra of the twisted  $C^*$ -algebra of  $G$ . We then describe (in terms of  $G$  and  $S$ ) the so-called Weyl groupoid and twist that J. Renault defined in 2008, which give us a different groupoid model for our Cartan pair. Time permitting, we discuss applications to higher rank graph  $C^*$ -algebras. This is joint work with A. Duwenig, E. Gillaspy, S. Reznikoff, and S. Wright.

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## An invitation to mean dimension of a dynamical system and the radius of comparison of the crossed product

N. Christopher Phillips

The purpose of this talk is to explain the background (including the terms below) and some progress towards the following conjecture, relating topological dynamics to the structure of the crossed product  $C^*$ -algebra.

Let  $G$  be a countable amenable group, let  $X$  be a compact metrizable space, and let  $h$  be an action of  $G$  on  $X$ . The mean dimension  $\text{mdim}(h)$  is a purely dynamical invariant, designed so that the mean dimension of the shift on  $([0, 1]^d)^G$  is equal to  $d$ . The radius of comparison  $\text{rc}(A)$  of a unital  $C^*$ -algebra  $A$  is a numerical measure of failure of comparison in the Cuntz semigroup of  $A$ , a generalization of unstable K-theory. It was introduced to distinguish  $C^*$ -algebras having no connection with dynamics. The conjecture asserts that if  $h$  is free and minimal, then  $\text{rc}(C^*(G, X, h)) = \frac{1}{2}\text{mdim}(h)$ . The inequality  $\text{rc}(C^*(G, X, h)) \leq \frac{1}{2}\text{mdim}(h)$  is known for  $G = \mathbb{Z}^n$ , and progress towards the inequality  $\text{rc}(C^*(G, X, h)) \geq \frac{1}{2}\text{mdim}(h)$  has been made for the known classes of examples of free minimal actions with nonzero mean dimension, for any countable amenable group  $G$ .

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# Unbounded Fredholm operators

Marina Prokhorova

A linear operator (bounded or unbounded) is called Fredholm if its range is closed and its kernel and cokernel are finite-dimensional.

The index theory for norm continuous families of bounded Fredholm operators was developed in the classical work of Atiyah; its analog for self-adjoint operators was developed in the work of Atiyah and Singer. The index theory of elliptic differential operators on closed manifolds is based on these classical results.

However, in some situations (e.g., for elliptic operators on manifolds with boundary) one needs to deal with weaker topologies on the space of unbounded operators. The most important such topology is the graph topology; a family of operators is continuous in the graph topology if the family of their graphs is continuous.

My talk is devoted to an index theory of graph continuous families of unbounded Fredholm operators in a Hilbert space. I will show how this theory is related to the classical index theory of bounded Fredholm operators. The talk is based on my recent preprint arXiv:2110.14359

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## Abelian twisted groupoid extensions

Jean Renault

I will present a variant of Mackey normal subgroup analysis. In presence of a normal subgroupoid over which the twist is abelian, the  $C^*$ -algebra of a twisted locally compact groupoid with Haar system can be decomposed via an equivariant Gelfand isomorphism onto another twisted groupoid  $C^*$ -algebra. I will give two applications, the first one to  $C^*$ -algebraic deformation quantization and the second to multiplier representations of locally compact abelian groups.

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## Stability of Cuntz–Krieger relations

Guy Salomon

Let  $S$  be a set of noncommutative continuous functions on  $d$  variables, for example, free polynomials on  $d$  variables and their adjoints, defined on some noncommutative set, for example  $\bigsqcup_n M_n(\mathbb{C})^d$ . Then  $S$  is called *stable* if for every  $\epsilon > 0$  there is  $\delta > 0$  such every  $d$ -tuple, on which  $S$  vanishes up to  $\delta$ , is  $\epsilon$ -close to some other  $d$ -tuple on which  $S$  truly vanishes. Here the words “up to” and “close to” obviously depend on a fixed metric. For example, it is well-known that with respect to the operator norm, the polynomial sets  $\{z\bar{z} - 1, \bar{z}z - 1\}$  and  $\{z^2 - z, z - \bar{z}\}$  are stable.

Stability of two particular families of polynomial sets — relations defining a group and Cuntz–Krieger relations of a directed graph — is related to other areas of mathematics. In this talk, I will describe some connections to computer science and group theory when the norm is either the operator norm or the Hilbert–Schmidt norm. I will then focus on Cuntz–Krieger relations and characterize those graphs whose relations are stable in the Hilbert–Schmidt norm.

The talk is based on ongoing work with Omer Lavi.

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# Quotient modules of the Hardy module on the polydisc

Jaydeb Sarkar

Here the Hardy module refers to the Hardy space over the open unit  $n$ -polydisc, and quotient modules are closed subspaces of the Hardy module that are jointly co-invariant under the multiplication operators by the coordinate functions. In this talk, we will briefly recall the structure of quotient modules of the one variable Hardy module (namely, the classical Beurling theorem). Then we will move on to  $n > 1$  case, and explain the difficulties, known, and unknown, along with some recent progress. Part of this talk is based on joint work with R. Debnath and B. Das.

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## Tilings, $K$ -theory, and Cohomology

Chaim (aka Claude) Schochet

When I was a grad student in algebraic topology at the U. of Chicago, I was taught that algebraic topologists had the right to invade other areas of mathematics (differential geometry, pure algebra, ...) and pillage, proving results there that were not known to the experts in those fields. (The best-known recent example is Gunnar Carlsson's invention and use of topological data analysis to invade biology and medicine.) Having shifted to  $C^*$ -algebras, I have found this attitude to be very useful.

Eric Akkermans, a physicist at the Technion, asked for some help in understanding a mysterious relationship observed in the laboratory that arose in the study of aperiodic tilings (google "Penrose tiles", for instance, and look at the images) between diffraction patterns (Bragg peaks) of a tiling and the spectral properties of the Hamiltonians of the tilings. Working with Jon Rosenberg and Eric's student Y. Don, we found that they are related via the index theorem for foliated spaces, which is the  $C^*$ -algebra generalization of the Atiyah-Singer index theorem.

My talk will focus on the  $K$ -theory and cohomology part of this story: I will state one theorem, put one diagram on the board, and I will spend the hour explaining what the spaces, the algebras, the groups and the measures mean, and why the diagram commutes. I promise not to prove anything. If you're really curious, though, check out

E. Akkermans, Y. Don, J. Rosenberg, C. Schochet, *Relating diffraction and spectral data of aperiodic tilings: towards a Bloch theorem*, J. Geom. Phys. 165 (2021) 104217.

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## **C\*-envelopes of tensor algebras of product systems**

Camila Sehnem

The C\*-envelope of an operator algebra  $C$  is the smallest C\*-algebra generated by a completely isometric copy of  $C$ . Muhly and Solel showed that the C\*-envelope of the tensor algebra of a correspondence  $\mathcal{E}$  is canonically isomorphic to its Cuntz-Pimsner algebra under certain assumptions on  $\mathcal{E}$ , which were later removed by Katsoulis and Kribs. In this talk I will report on a generalisation of this result for an arbitrary product system  $(\mathcal{E}_p)_{p \in P}$  over a submonoid  $P$  of a group  $G$ . The main theorem implies that the C\*-envelope of the tensor algebra of  $(\mathcal{E}_p)_{p \in P}$  automatically carries a generalised gauge coaction of  $G$ , answering a question left open in recent work of Dor-On, Kakariadis, Katsoulis, Laca and Li.

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## **Noncommutative rational functions on the unit ball**

Eli Shamovich

In this talk I will present joint work with Michael Jury and Robert Martin. I will discuss noncommutative (nc) rational functions that contain the unit row ball in their domain and that are bounded on it. Such nc rational functions define bounded left multipliers on the full Fock space. We will discuss realizations of such functions, their inner-outer decomposition, and the corresponding Clark measures. We will demonstrate the connection between finitely-correlated representations of the Cuntz algebra and nc rational inner functions based on the work of Davidson, Kribbs, and Shpigel.

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## **Noncommutative numerable principal bundles from group actions on C\*-algebras**

Mariusz Tobolski

The notion of a compact noncommutative (or quantum) principal bundle, which generalizes the Cartan compact principal bundle from topology (local triviality not assumed), emerged in the literature almost 30 years ago. Recently, the difficulty of introducing the local-triviality condition to the noncommutative realm was overcome using the notion of the local-triviality dimension of an action of a compact quantum group on a unital C\*-algebra. In this talk, I will propose a definition of a locally trivial noncommutative principal bundle in the setting of actions of locally compact Hausdorff groups on (possibly non-unital) C\*-algebras. I will discuss various motivations and technical difficulties that appear in the non-compact case. I will also provide some basic results and examples. The key difference is that, although the problem itself can be described in the language of C\*-algebra, one is quickly led beyond the Gelfand-Naimark duality and to the theory of multipliers of the Pedersen ideal.

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# Lévy processes on quantum groups and examples

Ami Viselter

We will introduce Lévy processes and locally compact quantum groups, show how the former can be defined on the latter, and give examples. We will focus on quantum groups arising as Rieffel deformations of groups. Based on joint work with Adam Skalski.

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