Challenges in Probability and Statistical Mechanics

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Titles and Abstracts

Metric graph extensions of lattice models with applications in stat mech and quantum systems

Michael Aizenman

Princeton

As a counterpoint to "be wise and discretize", continuous extensions are relevant and provide useful perspective. They occasionally pose challenges but also yield new tools. Examples of both may be found in: the contact process as extension of discrete percolation, long-range 1D Ising and percolation models, the quantum Ising model, quantum spin chains, influence propagation in the random-field Ising model estimated through a stopping time argument, extensions of discrete random height functions, and new results for the Villain $O(2)$ spin system though its metric graph representation.

Graphical representations and loop $O(1)$ as factors of i.i.d.

Omer Angel

UBC

The Loop $O(1)$ model comes up in graphical representations of the Ising model, and is also of interest in its own right. We show that in several broad cases the loop $O(1)$ model can be represented as a factor of an i.i.d. process.

A new variational condition for uniqueness of Doeblin measures

Noam Berger
TU Munich

g-functions and g-measures were introduced by Doeblin and Fortet in 1937 as a generalisation of Markov chain. A g-function is a (continuous) function from all (one - sided) infinite words in a certain alphabet to the space of probability measures on the same alphabet. A g-measure for a given g-function is a translation invariant measure on two-sided sequence in the alphabet, such that the distribution of the symbol at 0 conditioned on the entire history is the value that the g-function gives to this history. Doeblin and Fortet asked under which conditions a g-measure is unique, and provided a sufficient condition in terms of the variation of the g functions. Later progress included examples of g-functions which possess multiple g-measures, as well as new (and weaker) conditions for uniqueness.

Our goal in this talk is threefold:

1. Present a new variational condition for uniqueness, which we believe is sharp.

2. Show that a minor modification of a recent example for non-uniqueness of g-measures serves as a counter example to a conjecture of Walters (circa 1960).

3. Replace the confusing terminology g-function and g-measure by new terminology. We suggest Doeblin functions and Doeblin measures.

Based on joint work with Diana Conache, Anders Oberg and Anders Johannson.

Extremal sets for two-dimensional random walks

Marek Biskup
UCLA

I will discuss the large/small scale structure of certain natural extremal sets for planar random walks on finite domains with wired boundary condition run for times proportional to the cover time of the resulting finite graph. The main focus will be on the points avoided by the walk up to that time (a.k.a. late points) as well as the thick and thin points. As it turns out, these sets admit scaling limits that are close, but not exactly equal, to the Liouville Quantum Gravity measure which is the scaling limit of the thick points of the Discrete Gaussian Free Field. The connection between the two settings is supplied by 2nd Ray-Knight Theorem. Time permitting I will also note the highlights of the recent derivation of the scaling limit of the time spent at the most visited vertex by the random walk on regular trees, addressing a question initiated by Erdős and Taylor in 1960. Based on joint papers involving Y. Abe, S. Lee and O. Louidor.

Properties of the gradient squared of the Gaussian free field

Alessandra Cipriani

University College London

In this work we study the scaling limit of a random field which is a non-linear transformation of the gradient Gaussian free field. More precisely, this interface is the recentered square of the norm of the gradient Gaussian free field at every point of the square lattice. Surprisingly, in dimension 2 this field bears a very close connection to the height-one field of the Abelian sandpile model studied in Dürre (2009). In fact, with different methods we are able to obtain the same scaling limits of the height-one field: on the one hand, we show that the limiting cumulants are identical (up to a sign change) with the same conformally covariant property, and on the other that the same central limit theorem holds when we view the interface as a random distribution. We generalize these results to higher dimensions as well. Joint work with Rajat Subhra Hazra (Leiden), Alan Rapoport (Utrecht) and Wioletta Ruszel (Utrecht)

Coalescence of geodesics and the BKS midpoint problem in first-passage percolation

Barbara Dembin

ETH Zurich

We consider first-passage percolation on $\mathbb{Z}^2$ with independent and identically distributed weights. Under the assumption that the limit shape has at least 32 extreme points, we prove that geodesics with nearby starting and ending points have significant overlap, coalescing on all but small portions near their endpoints. The statement is quantified, with power-law dependence of the involved quantities on the length of the geodesics.

The result leads to a quantitative resolution of the Benjamini–Kalai–Schramm midpoint problem. It is shown that the probability that a geodesic passes through a given edge is smaller than a power of the distance between that edge and the endpoints of the geodesic.

Joint work with Dor Elboim and Ron Peled.

Sparse random graphs with unusually large subgraph counts

Amir Dembo
Stanford

In this talk, based on joint works with Nicholas Cook, Huy Tuan Pham and Sohom Bhattacharya, I will discuss recent developments in the emerging theory of nonlinear large deviations, focusing on sharp upper tails for counts of several fixed subgraphs in a large sparse random graph (such as Erdős–Rényi or uniformly d-regular). These results allow in turn to determine the typical structure of samples from an associated class of Gibbs measures, known as Exponential Random Graph Models, which are widely used in the analysis of social networks.

An isomorphism theorem for anharmonic fields and scaling limits

Jean-Dominique Deuschel
TU Berlin

We introduce a natural measure on bi-infinite random walk trajectories evolving in a time-dependent environment driven by the Langevin dynamics associated to a gradient Gibbs measure with convex potential. We derive an identity relating the occupation times of the Poissonian cloud induced by this measure to the square of the correspond- ing gradient field, which - generically - is not Gaussian. In the quadratic case, we recover a well-known generalization of the second Ray-Knight theorem. We further determine the scaling limits of the various objects involved in dimension 3, which are seen to exhibit homogenization. In particular, we prove that the renormalized square of the gradient field converges under appropriate rescaling to the Wick-ordered square of a Gaussian free field on $\mathbb{R}^3$ with suitable diffusion matrix, thus extending a celebrated result of Naddaf and Spencer regarding the scaling limit of the field itself. A joint work with Pierre-Francois Rodriguez

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Irreducibility of random polynomials

Gady Kozma
Weizmann Institute of Science

Take a random polynomial whose coefficients are 1 or -1 with equal probability and are independent. What is the probability that it is irreducible over the rationals, asymptotically as the degree goes to infinity? We will survey recent progress on this fascinating problem, which is still open. Joint work with Lior Bary-Soroker and Dimitris Koukoulopoulos.

Asymptotic expansions in spin $O(N)$ models via reflection positivity

Sébastien Ott
University of Fribourg

In the study of spin $O(N)$ models, a successful heuristic argument (vastly used in physics) is that the model is locally behaving like a Gaussian field at low enough temperature. This idea is usually referred to as the ”spin wave picture”. A way to make sense of that picture is to imagine that the spin $O(N)$ model is (at least formally) a Gaussian Free Field at 0 temperature, and that positive temperatures are a ”perturbation” of that Gaussian field. A useful conclusion of that idea is that one can (still formally) expand quantities of interest (like the free energy or correlation functions) as an asymptotic power series in T (the temperature) with the 0th coefficient being the Gaussian one. In this talk, I will first introduce the problem in more detail and present an approach based on reflection positivity to (rigorously) perform such expansions, therefore supporting the spin wave picture. The approach is initially due to Bricmont, Fontaine, Lebowitz, Lieb, and Spencer in the context of the XY model and was recently extended to general Ns (in dimension at least 3) in a joint work with A. Giuliani.

The scaling limit of the 2D discrete Gaussian model

Jiwoon Park
Cambridge

The discrete Gaussian model (or the integer-valued GFF) is a doubly discrete random field model imitating the Gaussian free field. Because of its relation to some fundamental problems in physics, such as $U(1)$ gauge field theory and the Kosterlitz-Thouless phase transition in XY model, this model had drawn the attention of a number of mathematical physicists. Despite the growing understanding of the KT phase recently, studying the exact limiting behaviour of related models often turn out to be challenging. I will briefly discuss how the renormalisation group argument can be used to construct the scaling limit of the generalised group of sine-Gordon type models, and how the 2D discrete Gaussian model can be reduced to the study of this. Some open problems and possible approaches will also be discussed.

T.B.A.

Gábor Pete
Alfréd Rényi Institute of Mathematics & Budapest University of Technology and Economics

T.B.A.

Hastings-Levitov in a cylinder

Eviatar Procaccia
Technion - I.I.T.

We construct a Hastings-Levitov (0) process in finite cylinders and show that under appropriate scaling of the slit sizes as a function of the length of the cylinder, this process converges to the Stationary Hastings Levitov (0). Moreover, like in the infinite stationary case, we can show that particle sizes are tight, suggesting that this models diffusion limited aggregation on a cylinder, thus allowing us to calculate exact growth rate and the time the aggregate collapses to a single arm.

This reports on work in progress with Anna Zuchenko.

Critical exponents for three-dimensional percolation models with long-range dependence

Pierre-François Rodriguez
Imperial College London

the talk will report on recent progress concerning the near-critical behaviour of certain percolation models in three dimensions. The results deal with the phase transition associated to two related percolation problems involving the Gaussian free field (GFF) in 3D. In one of these cases, they determine a unique "fixed point" associated to the transition, comprising a set of exponents which are proved to obey Fisher's scaling law. This is one of several relations classically conjectured by physicists to hold on the grounds of a corresponding scaling ansatz.

Random field induced order in 2D

Wioletta Ruszel
University of Utrecht

In this talk we will discuss random field induced ordering. In particular we shall prove that a classical $O(2)$ model subjected to a weak i.i.d. Gaussian field pointing in a fixed direction exhibits residual magnetic order on the square lattice $\mathbb{Z}^2$ and moreover aligns perpendicular to the random field direction. This type of transition is also referred to as of spin-flop type.

Our approach is based on a multi-scale Peierls contour argument developed.

This is joint work with N.Crawford (Technion, Israel).

Dynamical localization for random band matrices up to $N^{1/4}$ band width

Jacob Shapiro

Princeton

We consider a large class of $N \times N$ Gaussian random band matrices with bandwidth W, and prove that for $W \ll N^{1/4}$ they exhibit Anderson localization at all energies. To prove this result, we rely on the fractional moment method, and on the so-called Mermin-Wagner shift (a common tool in statistical mechanics). Joint with Cipolloni, Peled, and Schenker.

A Tale of Three Coauthors: Comparison of Ising Models

Barry Simon

Caltech

On Friday, Jan 14, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three coauthors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

The hardcore model on a random subgraph of the hypercube

Yinon Spinka

UBC

The number of independent sets in the d-dimensional hypercube $\{0,1\}^d$ was estimated precisely by Korshunov and Sapozhenko in the 1980s and recently refined by Jenssen and Perkins. In this talk we discuss new results on the number of independent sets in a random subgraph of the hypercube. Similar results are also obtained for the partition function of the hardcore model at various fugacities. The results rely on an analysis of the antiferromagnetic Ising model on the hypercube, which is based on various tools including cluster expansions and coarse-graining of contours. Joint work with Gal Kronenberg.

Noise sensitivity of percolation via differential inequalities

Vincent Tassion
ETH Zurich

In 1999, Benjamini, Kalai and Schramm proved that critical crossing probabilities for planar Bernoulli percolation are noise sensitive. Ten years later, Garban, Pete and Schramm obtained a sharp quantitative version of this result. We present a new proof of this result, inspired by Kesten's theory of scaling relations. As we will see in the talk, the proof relies on geometrical arguments and not on spectral methods (contrary to previous approaches).This talk is based on a joint work with Hugo Vanneuville.

Growth of Stationary Hastings-Levitov

Amanda Turner
University of Leeds

Planar random growth processes occur widely in the physical world. One of the most well-known, yet notoriously difficult, examples is diffusion-limited aggregation (DLA) which models mineral deposition. This process is usually initiated from a cluster containing a single "seed" particle, which successive particles then attach themselves to. However, physicists have also studied DLA seeded on a line segment. One approach to mathematically modelling planar random growth seeded from a single particle is to take the seed particle to be the unit disk and to represent the randomly growing clusters as compositions of conformal mappings of the exterior unit disk. In 1998, Hastings and Levitov proposed a family of models using this approach, which includes a version of DLA. In this talk I will define a stationary version of the Hastings-Levitov model by composing conformal mappings in the upper half-plane. This is proposed as a candidate for off-lattice DLA seeded on the line. We analytically derive various properties of this model and show that they agree with numerical experiments for DLA in the physics literature.

This talk is based on arXiv:2008.05792, which is joint work with Noam Berger and Eviatar Procaccia.

Quantum spin systems, their loop representations and extremal state decomposition

Daniel Ueltschi
University of Warwick

I will describe a family of $S = 1$ quantum spin systems with quadratic and bi-quadratic interactions in dimension $d = 3+$. Its phase diagram is interesting and includes ferromagnetic, antiferromagnetic, and nematic phases.

I will discuss two unproved conjectures:

1. the joint distribution of loop lengths is Poisson-Dirichlet;
2. some explicit extremal state decompositions for the nematic and $SU(3)$ phases.

It turns out that these conjectures are related.

I will review limited rigorous results, such as the existence of phase transitions using the method of reflection positivity, and calculations on the complete graph.

Large-distance behavior of non-critical Ising correlations. Known results and open problems

Yvan Velenik
University of Geneva

The large-distance asymptotic behavior of correlation functions (i.e., covariances of local random variables) in statistical mechanical systems have been investigated for more than a century, starting with the celebrated work of Ornstein and Zernike in 1914 and 1916. I'll review the state of the art in the context of the ferromagnetic Ising model on $\mathbb{Z}^d$, focusing mainly on rigorous non-perturbative results valid at all non-critical temperatures.

Perspectives and recent results on quantum spin glasses

Simone Warzel
TU Munich

Studying the effects of a transversal field on classical spin glass models is of interest in a variety of contexts ranging from quantum computing to the statistical mechanics describing the static or dynamic phase transitions. In this talk, I will give an overview over recent results concerning the free energy and other large-deviation functionals as well as spectral properties of the underlying random-matrix models.

Extremes for the Gaussian free field in random environment

Ofer Zeitouni
Weizmann Institute of Science

We consider logarithmically correlated Gaussian fields, indexed by $\mathbb{Z}^d$, where the correlation depends on a random environment and is not "uniformly logarithmic". We prove a general convergence result for the maximum, and apply it, in dimension 2, to the cases of GFF in random conductances and on the highly supercritical percolation cluster.

Joint work with Florian Schweiger
