How many degree d plane curves pass through $3d-1$ points?

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Enumerative geometry is one of the oldest disciplines within algebraic geometry, going back to Ancient Greece where the answer was already known to the simple question: how many lines go through two points in the plane? This is the $d = 1$ case of the question in the title, and with only basic linear algebra one can see that the answer to the $d = 2$ case is also one. But for larger degrees the question becomes increasingly subtle. Indeed, the $d = 3$ case was only resolved to be 12 by Steiner in 1848, and the $d = 4$ case was resolved in the 19th century by Zeuthen. By that time enumerative geometry had outstripped its rigorous foundations, and even when these were developed in the twentieth century, progress toward this question remained halted. It was only using ideas coming from the mirror symmetry phenomenon in string theory, that Kontsevich succeeded in giving a complete answer to this question. In this research group, we study some of the classical methods for solving these kinds of problems with a view toward problems on the modern frontiers of the subject.