

Title: On singularity properties of word maps and applications to random walks on compact p-adic groups.

Abstract: Given a word w in a free group F_r on a set of r elements, and a group G , one can associate a word map $w:G^r \rightarrow G$. When G is compact, w induces a natural probability measure on G , and one can study the corresponding random walk. We study the collection of random walks on $SL_n(\mathbb{Z}/p^k\mathbb{Z})$ induced by w , as p, k and n vary.

It turns out that various mixing properties of these random walks can be characterized by the geometry of the fibers of the word maps $w:SL_n(\mathbb{C})^r \rightarrow SL_n(\mathbb{C})$, and their concatenations (also called convolutions) $w^*w^*\dots^*w:SL_n(\mathbb{C})^r \rightarrow SL_n(\mathbb{C})$. When fixing $k=1$, and running over p and n , the corresponding random walks are essentially controlled by the dimensions of the fibers.

When running over p, n and k , the singularities of w come into play.

We show that word maps on semisimple Lie groups and Lie algebras have nice singularity properties after sufficiently many self-convolutions (with bounds depending only on the word). As a consequence, we obtain some uniform results on the above collection of random walks.

Based on a joint work with Yotam Hendel.