Noncommutative principal bundles from group actions on C*-algebras

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Noncommutative Analysis at the Technion
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Free actions of groups on spaces

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- A contractible free G-space EG can be used to define the equivariant cohomology of any G-space (Borel, 1959).
- Let G be a compact group and X is a compact G-space. Then X is free if and only if the natural map $K^*(X/G) \to K^*_G(X)$ (Atiyah and Segal, 1968).

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A G-action on A is *saturated* if a certain bimodule implements a Morita equivalence between the fixed-point subalgebra A^G (noncommutative analog of X/G) and the crossed product $A \rtimes G$.

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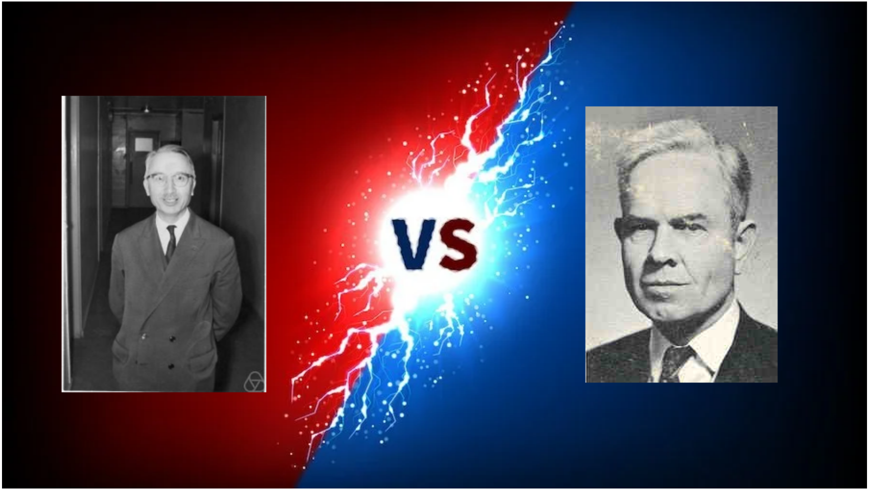
There are many other noncommutative generalizations of free actions:

- N. C. Phillips. Equivariant K-theory and freeness of group actions on C*-algebras, 1987.
- N. C. Phillips. Free actions of finite groups on C*-algebras, 2009.
- D. Ellwood. A new characterisation of principal actions, 2000.
- K. De Commer, M. Yamashita. A construction of finite index C*-algebra inclusions from free actions of compact quantum groups, 2013.

Two competing definitions of PBs in the 50's



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R. S. Palais, *On the existence of slices for actions of non-compact Lie groups*, Ann. of Math. (2) 73 1961 295–323.

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Right!

We say that A is *trivial* as a G-C*-algebra if there is a G-equivariant non-degenerate $*$-homomorphism $C_0(G) \to M(A)$.

A trivial example

Example

For a locally compact group G, let $\mathcal{K} := \mathcal{K}(L^2(G))$ be the C*-algebra of compact operators on $L^2(G)$ (where we take the Haar measure on G), and let $\lambda : G \to U(B(L^2(G))$ be the left-regular representation.

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Define a G-action on $\mathcal{K}$ by

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Define a G-action on $\mathcal{K}$ by

$$T \ \mapsto \ \lambda(g)T\lambda(g)^*, \qquad T \in \mathcal{K}, \quad g \in G.$$

Then there exists a non-degenerate G-equivariant $*$-homomorphism

$$C_0(G) \longrightarrow M(\mathcal{K}) \cong B(L^2(G)).$$

Classically local trivial G-C*-algebras (I)

Let $\pi : Y \to Y/G$ be a locally compact Steenrod principal G-bundle and let A be a G-C*-algebra. Let us use the concept of the $C_0(Y)$-algebra of Kasparov.

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Definition

We say that A is *classically locally trivial* as a G-C*-algebra if there exists a G-equivariant $*$-homomorphism $\chi : C_0(Y) \to Z(M(A))$ such that $\chi(C_0(Y))A$ is norm dense in A.

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Proposition

Let G be a locally compact group acting on a C-algebra A with the Hausdorff spectrum $\widehat{A}$. Then $\widehat{A} \to \widehat{A}/G$ is a Steenrod principal G-bundle if and only if A is a classically locally trivial G-C*-algebra.*

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- If Y and G are second countable and G is additionally torsion free, then classically locally trivial algebras are exactly the *proper algebras* of Guentner, Higson, and Trout, where the notion of proper action is meant in the sense of Baum, Connes, and Higson.

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- If Y and G are second countable and G is additionally torsion free, then classically locally trivial algebras are exactly the *proper algebras* of Guentner, Higson, and Trout, where the notion of proper action is meant in the sense of Baum, Connes, and Higson.
- Most trivial G-C*-algebras in the sense introduced during the talk are not classically locally trivial. Moreover, for any Steenrod principal G-bundle $Y \to Y/G$ there is a classically locally trivial G-C*-algebra $C_0(Y, \mathcal{K}(L^2(G)))$, which is in fact trivial as G-C*-algebras.

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- In the locally compact case the trivializing cover of X may have infinite cardinality, so there is a question of convergence of $\sum_i \sigma_i$. Moreover, the partition of unity functions $\sigma_i : X \to [0, 1]$ will only be bounded in this case. They are not compactly supported or vanishing at infinity.
- Numerable principal G-bundle are classified using the universal principal G-bundle EG which is not locally compact. Hence, we have to go beyond the Gelfand–Naimark duality.

Reformulation of local triviality (II)

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Theorem (MT)

$X \to X/G$ is a numerable Steenrod principal G-bundle if and only if there exist G-equivariant maps

$$\phi_i : X \to \mathcal{C}G, \qquad i \in \mathbb{N},$$

such that $\left(\sum_{i=0}^{n} \mathrm{t} \circ \phi_i\right)_n$ converges to 1 in the compact-open topology.

Compact case: the local-triviality dimension

Definition (E. Gardella, P. M. Hajac, MT, J. Wu)

Let G be a compact group and let A be a unital G-C*-algebra. The local-triviality dimension $\dim_{\mathrm{LT}}^{G}(A)$ is the minimal number n such that there exist G-equivariant unital $*$-homomorphisms

$$\rho_0, \ldots, \rho_n : C(\mathcal{C}G) \to A$$

satisfying $\sum_{i=0}^{d} \rho_i(\mathrm{t}) = 1$. We set $\dim_{\mathrm{LT}}^{G}(A) = \infty$ if no such n exists.

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Dimension zero = triviality

$\dim_{\mathrm{LT}}^{G}(A) = 0 \ \Rightarrow\ \exists\ G$-equivariant unital $*$-homomoprhism $C(G) \to A$.

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Theorem (Pedersen '66)

K_A is a two-sided, dense, order-related ideal in A, minimal among all such.

Multipliers of the Pedersen ideal (II)

By a (double) *multiplier* of K_A we mean a pair (S, T) of functions from K_A to K_A satisfying

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Let $\Gamma(K_A)$ denote the set of all multipliers of K_A.

One can show that $\Gamma(K_A)$ is a unital $*$-algebra and that $M(A) \subseteq \Gamma(K_A)$. The following seminorms

$$(S,T) \mapsto \|S(k)\|, \qquad (S,T) \mapsto \|T(k)\|, \qquad k \in K_A\,,$$

define the κ-topology on $\Gamma(K_A)$ (Lazar–Taylor '76).

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Theorem (Pedersen, Lazar–Taylor)

Let $A = C_0(X)$ for some locally compact space X. Then

$$K_A = C_c(X), \qquad \Gamma(K_A) = C(X),$$

and the κ-topology agrees with the compact-open topology.

Finally! Locally trivial G-C*-algebra

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Definition (MT)

We say that A is *locally trivial* as a G-C*-algebra if there exist G-equivariant unital continuous $*$-homomorphisms

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The classical case

Let G be a locally compact group and let A be a commutative G-C*-algebra. Then $A = C_0(X)$ for some locally compact Hausdorff G-space X.

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A is locally trivial as a G-C-algebra if and only if $X \to X/G$ is a numerable Steenrod principal G-bundle.*

Since $\Gamma(K_A) = C(X)$ for $A = C_0(X)$ and the κ-topology coincides with the compact-open topology, the result follows from the our reformulation of Steenrod principal G-bundles.

Local triviality implies classical local triviality

Lemma (MT)

Let $\chi : C_0(X) \to Z(M(A))$ be a G-equivariant $*$-homomorphism such that $\chi(C_0(X))A$ is norm dense in A. Then χ extends to a G-equivariant unital $*$-homomorphism $C(X) \to \Gamma(K_A)$, which is continous with respect to the compact-open topology on $C(X)$ and the κ-topology on $\Gamma(K_A)$.

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Proposition (MT)

Every classically locally trivial G-C-algebra is locally trivial.*

The local-triviality dimension

Let G be a compact group and let A be a unital G-C*-algebra.

If A has a unit, then $\Gamma(K_A) = A$ and the κ-topology coincides with the norm topology. Furthermore, if G is compact then so is $\mathcal{C}G$. Hence $C(\mathcal{C}G)$ is a unital C*-algebra and the compact-open topology on it coincides with the norm topology.

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If $\dim_{\mathrm{LT}}^{G}(A) < \infty$, then A is locally trivial as a G-C-algebra.*

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Proposition (MT)

If $\dim_{\mathrm{LT}}^{G}(A) < \infty$, then A is locally trivial as a G-C-algebra.*

Question: Can we prove that for unital locally trivial G-C*-algebras the family of $*$-homomorphisms $\rho_i : C(\mathcal{C}G) \to A$ has to be finite?

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2. Let A and B be G-C*-algebras. What are the morphisms between A and B for which if A is locally trivial then B is also locally trivial.
3. More examples! Including the purely noncommutative ones.
4. Generalize all above to actions of locally compact quantum groups.

Thanks!