Sunday, February 05

10:30 - 11:30

**Speaker :** Andreas Strömbergsson

**Title :** Rational points on horospheres, and small solutions to linear congruences

**Abstract :** I will discuss a new proof of a result by Einsiedler, Mozes, Shah and Shapira (2016) on the asymptotic distribution of primitive rational points on expanding horospheres in $\mathrm{SL}(d,\mathbb{Z}) \setminus \mathrm{SL}(d,\mathbb{R})$. The new proof leads to an explicit rate of convergence; it makes use of recent bounds on "martix Kloosterman sums" due to Erdelyi and Toth, and Rogers integration formula from 1955. As an application, we also obtain a refinement of a result by Strömbergsson and Venkatesh (2005) on counting small solutions to a system of linear congruences. Joint work with Daniel El-Baz and Min Lee.

Sunday, February 05

11:45 - 12:45

**Speaker :** Andreas Wieser

**Title :** Equidistribution of subspaces and their shapes

**Abstract :** Given a k-dimensional rational subspace of n-dimensional Euclidean space, one may associate to it two shapes: the shape of the integer lattice in the subspace and in the orthogonal complement. Moreover, one may measure the arithmetic complexity of a rational subspace by its discriminant which is the square of the covolume of the integer lattice in the subspace. Following work of Maass, Roelcke, and Schmidt it is conjectured that the set of triples consisting of a subspace L and the two shapes is equidistributed in the appropriate product space when L varies with fixed discriminant D and D goes to infinity. In this talk, we first recall known results towards this conjecture and then discuss new work joint with Aka, Einsiedler, Luethi, and Michel in which an effective variant of the conjecture is established for most dimensions. The proof uses a bootstrapping technique based on effective mixing as well as a discrepancy trick and will be discussed in a model case.

Sunday, February 05

15:00 - 16:00

**Speaker :** Oleg N. German

**Title :** Klein polyhedra as a multidimensional generalisation of continued fractions

**Abstract :** We are going to discuss Klein polyhedra, one of the most natural multidimensional generalisations of continued fractions. We shall start with the simplest case, the two-dimensional one, describing the geometric construction bearing the name of Klein polygons, which is basically a geometric interpretation of regular continued fractions. Many classical statements concerning continued fractions admit nice geometric interpretation. Having Klein polyhedra as a multidimensional generalisation of Klein polygons, it is natural to expect that every classical statement concerning continued fractions can be generalised to an arbitrary dimension, this way or another. The aim of the talk is to give a survey of currently known facts about such generalisations. We shall discuss how to generalise Lagrange's theorem on continued fractions, the concept of bad-approximability, the irrationality measure of a number, and the property of a quadratic irrationality to have the continued fraction expansion with a symmetric period. The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/.

Sunday, February 05

16:15 - 17:15

**Speaker :** Alexey Ustinov

**Title :** On statistical properties of 3-dimensional Voronoi-Minkowski continued fractions

**Abstract :** There exist two geometric interpretations of classical continued fractions admitting a natural generalization to the multidimensional case. In one of these interpretations, which is due to Klein, a continued fraction is identified with the convex hull (the Klein polygon) of the set of integer lattice points belonging to two adjacent angles (1895–1896). The second interpretation, which was independently proposed by Voronoi and Minkowski, is based on local minima of lattices, minimal systems, and extremal parallelepipeds (1896). The vertices of Klein polygons in plane lattices can be identified with local minima; however, beginning with the dimension 3, the Klein and Voronoi-Minkowski geometric constructions become different. The constructions of Voronoi and Minkowski is simpler from the computational point of view. In particular, they make it possible to design efficient algorithms for determining fundamental units in cubic fields. In both Voronoi's and Minkowski's approaches, the three-dimensional theory of continued fractions is based on interesting theorems of the geometry of numbers. Analytical approach based on the method of trigonometric sums and estimates of Kloosterman sums allows to solve different problems concerned with classical continued fractions. The talk will be devoted to analogous 3D tool. It is also based on the estimates of Kloosterman sums and uses Linnik-Skubenko ideas from their work "Asymptotic distribution of integral matrices of third order" (1964). This tool, in particular, allows to study statistical properties of Minkovski-Voronoi 3D continued fractions. The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

Monday, February 06

09:00 - 10:00

**Speaker :** Yitwah Cheung

**Title :** Perez Marco condition for simultaneous approximation

**Abstract :** Title: Perez Marco condition for simultaneous approximation Abstract: An irrational number satisfies Perez Marco condition if the infinite series $\sum \frac{\log\log q_{k+1}}{q_k}$ formed using the denominators of its convergents is summable. Consider the family of straight-line foliations of the genus two surface that is obtained by gluing together two copies of the standard flat torus along an embedded horizontal slit. In 2011, Cheung-Hubert-Masur established a dichotomy result for the Hausdorff dimension of the set of directions that give rise to non ergodic foliations according to whether or not the length of the slit satisfies the condition of Perez Marco. In this talk, I will discuss work-in-progress to extend this dichotomy result to the general case where the slit need not be horizontal. The new ideas needed for the extension involve two different generalisations of the best approximation theory of continued fractions.

Monday, February 06

10:30 - 11:30

**Speaker :** Nicolas Chevallier

**Title :** Doeblin-Lenstra conjecture and Legendre's theorem

**Abstract :** The Doeblin Lenstra conjecture asserts that for almost any real number $x$, the sequence $q_n|q_nx - p_n|$ is equidistributed with respect to a measure admitting a density. Moreover, this density is constant on the interval $[0, 1/2]$ and strictly smaller after $1/2$. It turns out that $1/2$ is also the largest possible constant in Legendre's theorem. Nakada proved that this phenomenon is valid for other continued fraction expansions. Continuing the work of Nakada, we study the relationship between the best constant in Legendre's theorem and the Doeblin-Lenstra conjecture two cases, the best approximations of a complex number by quotients of Gaussian integers and the best simultaneous approximations.

Monday, February 06

11:45 - 12:45

**Speaker :** Dzmitry Badziahin

**Title :** On effecive irratonality exponents of cubic irrationals

**Abstract :** Thanks to the remarkable work of Roth, it is known that the irrationality exponents of all irrational algebraic numbers equal two, i.e. they are the smallest possible. However, that result is ineffecive and hence it only gives that for algebraic irrational $x$ and $\lambda > 2$ the inequality $|x-p/q| < q^{-\lambda}$ has finitely many solutons but does not give a method for finding all of them. This ineffectiveness does not allow to use Roth's theorem in many applictiaons, in particular for Thue equations of degree at least 3. After Roth, the problem of estimating the effective irrationality exponents of algebraic irrationals attracted many mathematicians. In this talk we will show how recently discovered continued fractons of cubic irrationals can be applied to obtain non-trivial upper bounds of the effective irrationality exponents for a large class of cubic irrationals. In particular, for $x = (1+a)^{1/3}$, they give the same bounds as the classical hypergeometric method of Baker. On the other hand, they also provide non-trivial bounds for solutions of the equation $x^3 - ax^2 - t = 0$ for $|t| > 19.71a^{4/3}$. That is currently the best known result of this kind.

Monday, February 06

15:00 - 16:00

**Speaker :** Arturas Dubickas

**Title :** Distribution of the fractional and integral parts of geometric progressions

**Abstract :** Distribution of the fractional and integral parts of geometric progressions Let $\xi \neq 0$ and $\alpha > 1$ be two real numbers. We review some results concerning the distribution of the sequence of fractional parts $\{\xi\alpha^n\}, n = 0, 1, 2, 3, ...,$ and the sequence of integral parts $\lfloor \xi\alpha^n \rfloor, n = 0, 1, 2, 3, ....$ In the case of the first sequence (fractional parts), the problem is to show that it has "many" limit points and derive a lower bound between the largest and the smallest of those limit points. In the case of the second sequence (integral parts), it is of interest to investigate its divisibility properties. There is almost no progress in both problems for transcendental numbers $\alpha$ , so we only consider algebraic numbers $\alpha$. Some aspects of the second problem are of interest already for rational numbers $\alpha > 1$ and, in particular, for positive integers $b > 1$. For instance, for $b \in \{2, 3, 4, 6\}$ there is a finite set of prime numbers $S = S(b)$ such that for each real $\xi$ the sequence of integral parts $\lfloor \xi b^n \rfloor, n = 0, 1, 2, 3, ...,$ contains infinitely many elements divisible by at least one element of the set $S$. It seems likely that those four integers are the only integers greater than one with this property. To this end we prove that for "many" integers $b$ from the set $\{5, 7, 8, 9, 10, ...\}$ the opposite holds, namely, for any finite set of primes $S$ there is a real number $\xi > 0$ such that no element of the sequence $\lfloor \xi b^n \rfloor, n = 0, 1, 2, 3, ...,$ is divisible by a prime number from $S$.

Monday, February 06

16:15 - 17:15

**Speaker :** Nicolas de Saxcé

**Title :** Diophantine approximation in flag varieties

**Abstract :** The goal of the talk will be to present a general theory of Diophantine approximation in flag varieties, that are projective varieties on which some semisimple algebraic group acts transitively. We shall first explain how the methods of homogeneous dynamics apply to this problem, and then detail an application to some problems of Schmidt on rational approximations to linear subspaces.

Tuesday, February 07

09:00 - 10:00

**Speaker :** Yann Bugeaud

**Title :** Multiplicative rational p-adic approximation

**Abstract :**. Let $p$ be a prime number and $\xi$ an irrational p-adic number. Let $|\cdot|_p$ denote the p-adic absolute value normalized such that $|p|_p = p - 1$. The pigeon-hole principle shows that there exist $c > 0$ and, for every $A \geq 1$, integers $a$ and $b$ such that $0 < |ab|^{1/2} \leq A$ and $|b\xi - a|_p < cA^{-2}$. This result suggests the following problem: Does there exist $\xi$ such that

$$\inf_{a,b \neq 0} |ab| \cdot |b\xi - a|_p > 0?$$

This question, which we will discuss only briefly, has a similar flavour as the famous Littlewood conjecture in simultaneous approximation. A second question deals with p-adic numbers that are approximable by rationals at an order greater than 2. The multiplicative irrationality exponent $\mu^{\times}(\xi)$ (resp., uniform multiplicative irrationality exponent $\hat{\mu}^{\times}(\xi)$) of $\xi$ is the supremum of the real numbers $\mu$ for which the inequalities

$$0 < |ab|^{1/2} \leq A, \quad |b\xi - a|_p < A^{-\mu}$$

have a solution in integers $a, b$ for arbitrarily large real numbers $A$ (resp., for every sufficiently large real number $A$). We show that these exponents of approximation can be expressed in terms of exponents of approximation attached to a sequence of rational numbers defined from the Hensel expansion of $\xi$. We discuss the set of values taken by these exponents and compute them at the Thue-Morse p-adic number.

Tuesday, February 07

10:30 - 11:30

**Speaker :** Boris Adamczewski

**Title :**. Furstenberg's conjecture, Mahler's method, and finite automata

**Abstract :** It is commonly expected that expansions of numbers in multiplicatively independent bases, such as 2 and 10, should have no common structure. However, it seems extraordinarily difficult to confirm this naive heuristic principle in some way or another. In the late 1960s, Furstenberg suggested a series of conjectures, which became famous and aim to capture this heuristic. The work I will discuss in this talk is motivated by one of these conjectures. Despite recent remarkable progress by Shmerkin and Wu, it remains totally out of reach of the current methods. While Furstenberg's conjectures take place in a dynamical setting, I will use instead the language of automata theory to formulate some related problems that formalize and express in a different way the same general heuristic. I will explain how the latter can be solved thanks to some recent advances in Mahler's method; a method in transcendental number theory initiated by Mahler at the end of the 1920s. This a joint work with Colin Faverjon.

Tuesday, February 07

11:45 - 12:45

**Speaker :** Omri Solan

**Title :** Limits of compact diagonal orbits in the lattice space

**Abstract :** The diagonal subgroup $A < \mathrm{SL}_n(\mathbb{R})$ acts on the space of lattices. The compact $A$-orbits are classified using a number theoretic construction. We will discuss the following limit phenomenon: What are the possible limits of $A$-invariant measures on compact $A$-orbits? We prove that any ergodic A-invariant measure can rise as an ergodic component. To this end, we will use both dynamical properties of Hecke operators and a number theoretic construction. The talk is based on a joint work with Yuval Yifrach.

Tuesday, February 07

15:00 - 16:00

**Speaker :** Dmitry Kleinbock

**Title :** Dimension drop conjecture in homogeneous dynamics and applications to improvement of Dirichlet's Theorem

**Abstract :** Let $(X, \mu, T)$ be an ergodic probability measure preserving system on a metric space $X$, and let $U$ be a non-empty open subset of $X$. Consider the ($\mu$-null) set of points in $X$ whose trajectory misses $U$. When can one prove that this exceptional set has Hausdorff dimension less than the dimension of $X$? This dimension drop phenomenon has been conjectured for actions on homogeneous spaces and proved in several special cases, for example when $X$ is compact or has rank one. I will sketch a proof of the fairly general case of the conjecture - for arbitrary Ad-diagonalizable flows on irreducible quotients of semisimple Lie groups. Two main ingredients of the proof are effective mixing and the method of integral inequalities for height functions on $X$. The problem is connected to Diophantine approximation through the Dani Correspondence, and the talk will contain some motivating examples related to badly approximable systems of linear forms and improvement of Dirichlet's Theorem. Joint work with Shahriar Mirzadeh.

Tuesday, February 07

16:15 - 17:15

**Speaker :** Ilya Shkredov

**Title :** On Korobov bound concerning Zaremba's conjecture

**Abstract :** We prove in particular that for any sufficiently large prime $p$ there is $1 \leq a < p$ such that all partial quotients of $a/p$ are bounded by $O(\log p/\log\log p)$. This improves the well–known Korobov bound concerning Zaremba's conjecture from the theory of continued fractions. (Joint work with N.G. Moshchevitin and B. Murphy.)

Wednesday, February 08

09:00 - 10:00

**Speaker :** Elon Lindenstrauss

**Title :** Gap statistics in the spectrum of a flat torus

**Abstract :** J/W with Amir Mohammadi, Zhiren Wang Using recent advances in the study of unipotent flows, we establish quantitatively that for a flat torus satisfying a diophantine condition the pair correlation statistics for the spectrum conform to the expected Poisson behaviour of the spectrum. This gives an effective version of a qualitative result by Eskin, Margulis and Mozes.

Wednesday, February 08

10:30 - 11:30

**Speaker :** Manfred Einsiedler

**Title :** Diophantine approximation and measure rigidity for $\mathrm{SL}_2$

**Abstract :** We will discuss two measure rigidity theorems on quotients arising from forms of $\mathrm{SL}_2$, two adelic corollaries, and two consequences for the approximation of an arbitrary real number by rationals. The new theorems are joint work with Elon Lindenstrauss.

Wednesday, February 08

11:45 - 12:15

**Speaker :** Nikita Shulga

**Title :** Minkowski question mark and folding lemma

**Abstract :** For the famous Minkowski question mark function $?(x)$ we consider its fixed points, i.e. solutions of the equation $?(x) = x$. It turns out that some non-trivial results about fixed points of this function can be derived from the folklore statement from continued fraction theory called Folding Lemma. Namely, we can bound the rate of growth of partial quotients of some fixed points. Using ideas similar to the Folding lemma, we can get more results about Minkowski question mark. For example, we can prove that the derivative of $?(?(x))$ at quadratic irrational numbers is always equal to 0.

Wednesday, February 08

12:25 - 12:55

**Speaker :** Christoph Aistleitner

**Title :** The distribution of partial quotients of reduced fractions with fixed denominator

**Abstract :** In this talk we consider the distribution of the partial quotients of fractions $a/N$, where the denominator $N$ is fixed and a runs through the set of all integers which are coprime with $N$. There method is very general an allows to cover many different statistics. In particular we obtain a concentration result for the sum of partial quotients, which matches the tail behaviour which is known under an extra averaging over the denominators $N$. Similar results are obtained for the distribution of the maximal partial quotient. This is joint work with Bence Borda and Manuel Hauke. Arxiv reference: arXiv:2210.14095.

Thursday, February 09

09:00 - 10:00

**Speaker :** Anish Ghosh

**Title :** Variations on the Jarnik-Besicovitch Theorem.

**Abstract :** I will discuss several variations of the famous Jarnik-Besicovitch theorem in Diophantine approximation, including sets of "Exact approximation" and their geometric avatars.

Thursday, February 09

10:30 - 11:30

**Speaker :** Jörg Thuswaldner

**Title :** Multidimensional continued fractions and symbolic codings of toral translations A

**Abstract :** Sturmian sequence is an infinite string over two letters with subword complexity $p(n) = n + 1$. Morse and Hedlund (1940) as well as Coven and Hedlund (1973) established a surprising correspondence between Sturmian sequences and translations by an irrational number on the 1-torus $\mathbb{T}^1 = \mathbb{R}/\mathbb{Z}$. In their proof "balance properties" of Sturmian sequences play a prominent role. Several decades later, Arnoux and Rauzy (1991) observed that an induction process, in which the classical continued fraction algorithm appears, can be used to give another very elegant proof of this correspondence. In this talk we generalise this correspondence to higher dimensions. In particular, we show how multidimensional continued fraction algorithms can be used to obtain symbolic codings of low complexity for translations on the $d$-torus $\mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$. This is joint work with Valerie Berthe and Wolfgang Steiner.

Thursday, February 09

11:45 - 12:45

**Speaker :** Johannes Schleischitz

**Title :** On the Dirichlet spectrum and a "folklore set" in Diophantine approximation

**Abstract :** The so-called Dirichlet spectrum in Euclidean spaces of any dimension at least two, with respect to simultaneous approximation as well as for a linear form, is the entire interval $[0, 1]$. At least for simultaneous approximation, the same result still holds when restricting to certain classes of fractals. Moreover, the "folklore set" of vectors that are Dirichlet improvable but neither badly approximable nor singular has rather large Hausdorff and packing dimension. Some ideas of the constructive proofs of these facts are provided in the talk.

Thursday, February 09

15:00 - 16:00

**Speaker :** Alexander Gorodnik

**Title :** Density and Distribution of dense lattices

**Abstract :** We discuss distribution of of dense lattice projections in semisimple groups and connect this problem with analysis of averaging operators and spectral decomposition of unitary representations on homogeneous spaces.