SATELLITE MINI-WORKSHOP
Friday, February 10 & Sunday, February 12
Room 814, Amado Mathematics Building

Friday, February 10

09:00 - 09:20

Speaker : Alexey Ustionv

Title : Tropical Somos sequences

Abstract : Somos–(4) sequence $\{s_n\}$ is defined by initial data

$$s_1 = s_2 = s_3 = s_4 = 1$$

and recurrence relation

$$s_{n+2}s_{n-2} = s_{n+1}s_{n-1} + s_n^2.$$

It begins with

$$\ldots, 2, 1, 1, 1, 1, 2, 3, 7, 23, 59, 314, 1529, \ldots$$

In the talk we'll discuss a natural tropical generalization of this sequence and more general sequences of this type. The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

09:30 - 09:50

Speaker : Oleg German

Title : On uniform Diophantine exponents in multiplicative problems

Abstract : In classical Diophantine approximation there are two families of Diophantine exponents. They are called regular and uniform exponents. Given a setting, one of the main questions concerning the properties of Diophantine exponents is to describe their complete spectrum. It is well known that for one real number the uniform exponent is trivial. In this talk we shall explain how Davenport's lemma can help proving that uniform Diophantine exponents in multiplicative problems are also trivial.The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

10:00 - 10:20

Speaker : Alexander Kalmynin

Title : Values of cubic forms with non-typical binary expansions

Abstract : Certain properties of cyclotomic polynomials can be used to prove that any binary quadratic form admits infinitely many values at integer points which have large proportion of ones in binary expansion. In this talk, we will extend this result to some cubic forms and also discuss possible generalizations to forms of higher degree.The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

Friday, February 10

10:30 - 10:50

Speaker : Antoine Marnat

Title : Some open question about ordinary and uniform Diophantine approximation

Abstract : We will discuss relations between ordinary and uniform exponents of Diophantine Approximation and some open questions related to these objects.

Friday, February 10

11:00 - 11:20

Speaker : Dmirty Gayfulin

Title : On the derivative of the Minkowski question-mark function

Abstract :
Minkowski question-mark function $?(x)$ is a continuous monotonous function defined on $[0, 1]$ interval. If $[0; a_1, a_2, \ldots, a_n, \ldots]$ is the continued fraction expansion of an irrational number x, then

$$?(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{a_1+\ldots+a_k-1}}. \tag{1}$$

If x is a rational number, then (1) is replaced by finite sum. It is well known fact that the derivative of this function, if exists, can take only two values: 0 and $+\infty$. It is also known that the value of the derivative $?'(x)$ at the point $x = [0; a_1, a_2, \ldots, a_t, \ldots]$ is connected with the limit behavior of the sum $S_t(x) := a_1 + a_2 + \ldots + a_t$. We consider the following question: suppose that the derivative $?(x)$ exists and equals 0. What can one say about the sum $S_t(x)$? One can ask the same question in the situation when $?(x) = +\infty$. We will present some non-improvable results in this area.

Friday, February 10

11:30 - 11:50

Speaker : Taehyong Kim

Title : On a singular on average property

Abstract : The notion of singular on average was introduced by Kadyrov, Kleinbock, Lindenstrauss, and Margulis to generalize the notion of singular from a dynamical point of view. In this talk, we review the singular on average property and explain various properties related to this notion. In particular, we give equivalent conditions to the singular on average property using best approximations and inhomogeneous Diophantine approximation. This talk is based on a work with Wooyeon Kim and Seonhee Lim.

Friday, February 10

12:00 - 12:20

Speaker : Anurag Rao

Title : Dynamical questions arising from Dirichlet's theorem on Diophantine approximation

Abstract : Abstract - We study the notion of Dirichlet improvability in a variety of settings and make a comparison study between Dirichlet-improvable numbers and badly-approximable numbers as initiated by Davenport-Schmidt. The question we try to answer, in each of the settings, is – whether the set of badly-approximable numbers is contained in the set of Dirichlet-improvable numbers. We show how this translates into a question about the possible limit points of bounded orbits in the space of two-dimensional lattices under the diagonal flow. Our main result gives a construction of a full Hausdorff dimension set of lattices with bounded orbit and with a prescribed limit point.

Sunday, February 12

10:00 - 10:20

Speaker : Fedor Kuyanov

Title : Feynman checkers: number-theoretic properties

Abstract : My presentation is about Feynman checkers, an elementary model of electron motion introduced by R. Feynman. In this model, a checker moves on a checkerboard, and we count the turns. Feynman checkers are also known as a one-dimensional quantum walk. We prove some new number-theoretic results in this model, for example, sign alternation of the real and imaginary parts of the electron wave function in a specific area. All our results can be stated in terms of Young diagrams, namely, we compare the number of Young diagrams with an odd and an even number of steps. The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

Sunday, February 12

10:30 - 10:50

Speaker : Fedor Ozhegov

Title : Feynman checkers: external electromagnetic field and asymptotic properties

Abstract : Feynman checkers is one of the simplest models describing the motion of an electron. The model is also known as the one-dimensional quantum walk. We obtain an expression for the wave function of an electron moving in a homogeneous electromagnetic field in terms of hypergeometric functions and an asymptotic formula in terms of Bessel functions. The research is supported by grant of the Russian Science Foundation (project № 22-41-05001), https://rscf.ru/project/22-41-05001/

Sunday, February 12

11:00 - 11:20

Speaker : Mikhail Skopenkov

Title : Feynman checkers: quantum field theory on a checkered paper

Abstract : We present a new completely elementary model which describes creation, annihilation and motion of non-interacting electrons and positrons along a line. It is a modification of the model known under the names Feynman checkers, or one-dimensional quantum walk, or Ising model at imaginary temperature. The discrete model is consistent with the continuum quantum field theory, namely, reproduces the known expected charge density as the lattice step tends to zero. We show it using a number-theoretic toolbox. This is a joint work with Alexey Ustinov.

Sunday, February 12

11:30 - 11:50

Speaker : Athanasios Sourmelidis

Title : Restricted averages of Dedekind sums

Abstract : We investigate averages of Dedekind sums over rational numbers in $\mathcal{F} \cap [0, \alpha]$, where $\mathcal{F}$ denotes the set of Farey fractions and $\alpha \in [0, 1/2]$. We obtain asymptotics for the aforementioned averages when α is a rational number, confirming a conjecture of Ito, or an irrational number from a a full Lebesgue measure set. These results are tightly linked with the distribution of lengths of certain continued fraction expansions as well as the distribution of the involved partial quotients. In that regard we obtain a result quantifying the bias occurring in the second term of the asymptotic for the average running time of the *by-excess* Euclidean algorithm. This is joint work with Paolo Minelli and Marc Technau.

Sunday, February 12

12:00 - 12:20

Speaker : Manuel Hauke

Title : On the metric theory of approximations by reduced fractions: Quantifying the Duffin–Schaeffer conjecture

Abstract : Let $\psi : \mathbb{N} \to [0, 1/2]$ be given. Koukoulopoulos and Maynard (2020) proved the Duffin–Schaeffer conjecture: for almost all reals α there are infinitely many coprime solutions (p, q) to the inequality $|\alpha - p/q| < \psi(q)/q$, if and only if the series $\sum_{q=1}^{\infty} \varphi(q)\psi(q)/q$ is divergent. In a recent joint work with Christoph Aistleitner and Bence Borda, we established a quantitative version of this result in the following sense: for almost all α, the number of coprime solutions (p, q), subject to $q \leq Q$, is of asymptotic order $\Psi(Q) = \sum_{q=1}^{Q} 2\varphi(q)\psi(q)/q$.
In this talk, I will give an overview of the original proof of Koukoulopoulos and Maynard and the additional ideas we used to obtain this quantification.

Sunday, February 12

12:30 - 13:00

Discussion Session

Sunday, February 12

16:00 - 16:20

Speaker : Noy Soffer-Aranov

Title : Covering Radii of Lattices in Positive Characteristic

Abstract : A fascinating question in geometry of number pertains to the covering radius of lattice with respect to an interesting function. For example, given a convex body C and a lattice L in $\mathbb{R}^d$, it is interesting to ask what is the infimal $r \geq 0$ such that $L + rC = \mathbb{R}^d$. Another interesting covering radius is the multiplicative covering radius, which connects to dynamics due to its invariance under the diagonal group. It was conjectured by Minkowski that the multiplicative is bounded by above by 2^{-d} and that this upper bound is obtained only on $A\mathbb{Z}^d$. In joint work with Uri Shapira, we researched covering radii in the positive characteristic setting and discovered several surprising results. Some of our results include explicitly connecting between the covering radii with respect to convex bodies and successive minima and proving a positive characteristic analogue of Minkowski's function.

Sunday, February 12

16:30 - 16:50

Speaker : Agamemnon Zafeiropoulos

Title : Poissonian correlations: higher orders and weaker variants

Abstract : Let $(x_n)_{n\in\mathbb{N}} \subseteq [0,1]$ be sequence of points in the unit interval. We say that $(x_n)_{n\in\mathbb{N}}$ has Poissonian pair correlations (PPC) if

$$\lim_{N\to\infty} \frac{1}{N} \#\Big\{ m,n \leqslant N, m \neq n : \|x_m - x_n\| \leqslant \frac{s}{N} \Big\} = 2s \qquad \text{for all } s > 0.$$

It is known that sequences with PPC are also uniformly distributed. We show that the same conclusion is true for sequences with Poissonian correlations of any order $k \geqslant 3$. Moreover we define weaker variants of the notion of PPC and examine their relations with equidistribution. (Joint work with M. Hauke.)

Sunday, February 12

17:00 - 17:20

Speaker : Yuval Yifrach

Title : Equidistribution of Grids of Rings of Integers in Ranks 3,4,5

Abstract : Manjul Bhargava and Piper Harron proved that lattice shapes of rings of integers in ranks 3,4,5 become equidistributed when ordering the number fields by discriminant. We introduce a grid for any ring of integers in a number field, which records extra data compared to the shape. We will discuss our recent result on the equidistribution of those grids (also in ranks 3,4,5) in the space of grids. We will identify the additional data recorded in those grids and how to deduce Bhargava-Harron's result from our result.

Sunday, February 12

17:30 - 17:50

Speaker : Andrey Illarionov

Title : Korobov-Hlawka quadrature formulas

Abstract : Let N be a positive integer and $a_1, \dots, a_s$ be integers. To approximately evaluate multiple integrals with the help of quadrature formulas of functions on the s-dimensional unit cube, Korobov (1959) and Hlawka (1962) proposed using points of the form

$$x^{(k)} = \left(\left\{ \frac{a_1 k}{N} \right\}, \dots, \left\{ \frac{a_s k}{N} \right\} \right), \quad k = 1, \dots, N.$$

This idea gave rise to a whole direction on the borders of number theory and computational mathematics.
We will discuss some questions concerning with error of Korobov-Hlawka quadrature formulae.

Sunday, February 12

18:00 - 18:20

Speaker : Mordechay B. Levin

Title : On randomness at lattice points in parallelepipeds

Abstract : We will consider the Kesten discrepancy theorem and its generalizations.