Summer School 2023 on Algebraic Geometry:
Derived Categories, Stability Conditions, and Moduli
Abstracts

Izzet Coskun

Title: The geometry of moduli spaces of sheaves on surfaces

Abstract: I will discuss aspects of the geometry of moduli spaces of sheaves on surfaces, such as nonemptiness, irreducibility, topological and birational invariants and Brill-Noether loci.

Emanuele Macri

Title: Stable bundles on hyper-Kähler manifolds

Abstract: We survey recent advances in the theory of stable sheaves on hyper-Kähler manifolds as developed by O'Grady, Markman, and Beckman, generalizing to higher dimension Mukai's theory on K3 surfaces.

Paolo Stellari

Title: Geometric triangulated categories and exact functors: dg enhancements and weak approximation

Abstract: I will illustrate the most recent results on how to enhance triangulated categories and exact functors of geometric nature. Uniqueness of such enhancements and lifts will be discussed. We will then move to the problem of lifting equivalences between various triangulated categories and illustrate the new interplay between the theory of weakly approximable triangulated categories and the existing results about the uniqueness of enhancements. Applications to a generalization of a classical result by Rickard and to derived invariants of schemes will be discussed. The main results presented in the course are joint work (partly in progress) with Alberto Canonaco and Amnon Neeman.

Ed Segal

Title: GIT and derived equivalences

Abstract: This course will be an introduction to the theory of `window' subcategories, which are a tool for studying the derived categories of Geometric Invariant Theory quotients. We'll see how this theory can sometimes be used to prove that birational varieties have equivalent (or closely related) derived categories.

We'll start by working through some basic examples. We'll then sketch how the theory can be developed to cover harder examples, and discuss some recent general results. I'll assume

some background knowledge of derived categories of coherent sheaves but I won't assume any prior knowledge of GIT.

Anand Deopurkar

Title: Stability conditions, metrics, and compactifications

Abstract: A stability condition on a triangulated category turns the category into a geometric object. We can then import powerful techniques from geometry to the study of triangulated categories and stability manifolds. In these talks, I will elaborate on this theme. Specifically, I will describe how a stability condition defines a metric on the category. I will describe how this metric can be used to construct a compactification of the stability manifold, using ideas inspired by the study of metrics on Riemann surfaces, namely Teichmuller theory. I will then discuss potential applications, examples, and unexplored areas.

Nick Addington

Title: Induced maps on cohomology and topological K-theory

Abstract: A Fourier-Mukai kernel induces an exact functor between derived categories, and a compatible map on rational cohomology that doesn't respect the grading or the usual integral structure; but there is another integral structure that it does respect, coming from topological K-theory. After going through the basics, we will dig into examples including (as time permits) intersections of two quadrics, Calabi-Yau threefolds, Abelian varieties, K3 surfaces and cubic fourfolds, and P^n fibrations.