Attila Bérczes and Kálmán Győry

Effective results for Diophantine equations over finitely generated domains (I, II)

Abstract: There is an extensive literature of finiteness theorems for Diophantine equations with solutions from an integral domain finitely generated over $\mathbb{Z}$ which may contain transcendental elements, too. Most of these results are ineffective. Since the 1960s, a great number of ineffective finiteness theorems over number fields were made effective mainly by means of A. Baker's effective theory of logarithmic forms. Analogous results were established by Mason and others over function fields of characteristic 0 as well, providing effective bounds for the heights of the solutions. In the last decade Evertse and Győry extended an effective specialization method due to Győry from the 1980s to the case of arbitrary finitely generated domains of characteristic 0 over $\mathbb{Z}$. They reduced the initial equations over finitely generated domains to the number field and function field cases, and used the corresponding effective results over number fields and function fields to derive effective bounds for the solutions of the initial equations. Using this method, Bérczes, Evertse, Győry and Koymans, respectively, established general finiteness theorems over finitely generated domains of characteristic 0 for several classical equations, including unit equations in two unknowns, Thue equations, hyper- and superelliptic equations, Catalan equation and, recently, decomposable form equations and discriminant equations. In our talk we give a brief overview of the effective specialization method and the effective finiteness results obtained over finitely generated domains.

Pierre Dèbes

On the arithmetic of the ring of components of Hurwitz spaces

Abstract: Hilbert Irreducibility Theorem (HIT) allows a geometric approach of the Inverse Galois Problem, reducing it to finding rational points on Hurwitz moduli spaces of covers of the projective line. The talk will be devoted to the preliminary step of finding some irreducible component defined over the base field. Consideration of the so-called Harbater–Mumford components by Fried, Emsalem, Cau and the speaker led to first results. A next step forward was the introduction by Ellenberg–Venkatesh–Westerland of the "ring of components", which notably led them to a proof of the Cohen–Lenstra heuristics for rational function fields over finite fields. Conjoining various techniques with some HIT argument, Seguin managed to show arithmetic properties over number fields of the ring of components.

Julian Demeio

Weak weak approximation on Del Pezzo surfaces of low degree

Abstract: In recent joint work with Sam Streeter and Rosa Winter, we show that weak weak approximation holds for Del Pezzo surfaces of degree 2 (over a number field) with a rational point not lying on the ramification curve or on the intersection of 4 exceptional curves. To prove this, we use two geometric "procedures" to produce rational points on the surface. The points obtained by a certain iteration of these two procedures are parametrized by a rational higher-dimensional cover of the surface, and we deduce our result by proving the arithmetic surjectivity of the morphism defining the cover.

Alexei Entin

On the minimal ramification problem for the symmetric and alternating groups

Abstract: The minimal ramification problem asks for any given finite group G what is the minimal number $r(G)$ of primes (including infinity) that ramify in some extension of $\mathbb{Q}$ with Galois group G. In the case of the groups S_n and A_n, a conjecture of Boston and Markin predicts that $r(S_n) = r(A_n) = 1$. While this is still open, progress was made by Bary-Soroker and Schlank who were able to show that $r(S_n) \leq 4$ for all n. This result relies on the Green–Tao–Ziegler theorem on solutions in primes to linear systems of equations.

I will discuss further progress on this problem, showing that $r(S_n) \leq 3$ for n odd and that $r(A_n)$ is bounded provided $n \equiv 0, 1 \pmod 4$. Key ingredients in these results include the specialization method of Bary-Soroker and Schlank and recent advances on simultaneous prime values of a quadratic and linear form due to Lam, Schindler and Xiao.

I will give a survey of the minimal ramification problem and the Boston–Markin conjecture, discuss previous work on the subject (with emphasis on the work of Bary-Soroker and Schlank), state the new results and discuss the main ingredients of the proofs.

Arno Fehm

Rational points on ramified covers of abelian varieties over torsion fields

Abstract: The importance of Hilbert's irreducibility theorem led Colliot-Thélène, Sansuc and Serre to introduce the Hilbert property of varieties, which can be phrased either in terms of rational points or via specializations of covers. Later, Corvaja and Zannier proposed a less restrictive weak Hilbert property that holds for many more varieties and suffices in many applications. I will survey what is known about the stability of these two properties under geometric constructions and under base change. Then I will report on joint work with Bary-Soroker and Petersen in which we use the recent result by Corvaja, Demeio, Javanpeykar, Lombardo and Zannier for abelian varieties over finitely generated fields to deduce the weak Hilbert property for abelian varieties over certain infinite Galois extensions of number fields.

Saurabh Gosavi

Galois extensions of finite dimensional division algebras over function fields

Abstract: Given a finite group G, a G-Galois extension of a finite dimensional division algebra with center a field F is an extension of the form $D \otimes_F L/D$, where L/F is a G-Galois extension and $D \otimes_F L$ is a division algebra. In this talk, we will revisit the Brauer-Hilbertianity property due to Fein, Saltman and Schacher and as an application show that for function fields in one variable over number fields, groups that occur as Galois extensions of function fields can also be realized as Galois extensions of non-constant division algebras over such fields.

Damián Gvirtz

Hilbert Irreducibility Theorems for varieties of intermediate type

Abstract: I will survey recent results of mine (joint with Z. Huang/G. Mezzedimi) that establish versions of the Hilbert Property for many K3 surfaces and some higher-dimensional varieties of intermediate type.

David Hokken

Counting reciprocal Littlewood polynomials with square discriminant

Abstract: A Littlewood polynomial is a single-variable polynomial all of whose coefficients lie in ± 1. It is reciprocal if its list of coefficients forms a palindrome. We establish the leading term asymptotics of the number of reciprocal Littlewood polynomials with square discriminant. This relates to a bounded-height analogue of the Van der Waerden conjecture on Galois groups of random polynomials.

Joachim König

On exceptional local behavior of number fields and of rational functions

Abstract: We report on some recent results on field extensions, polynomials and rational functions whose local behavior qualifies them as "exceptional". Firstly, we investigate a notion of "fake subfields" of number fields, i.e., pairs of fields which look like "subfield and overfield" from a local point of view. Secondly, we investigate finite sets of rational functions over $\mathbb{Q}$ whose value sets, from a local point of view, seem to cover all of $\mathbb{Q}$. Both problems are connected via an innocent-looking group-theoretical property previously studied in the context of so-called intersective polynomials. I will put these problem in the context of related notions such as the classical notions of "arithmetically equivalent fields" and"exceptional polynomials", as well as more recent ones such as value multiplicity of polynomials.

Daniel Krashen

Brauertsch fields: specializations and local-global principles for the Brauer group

Abstract: In this talk, I'll describe some joint work with Max Lieblich and Minseon Shin concerning new local-to-global principles for Brauer classes over certain fields. These principles are derived from the notion of a Brauertsch field, a concept related to the "Brauer-Hilbertian" property of fields of Fein, Saltman and Schacher.

Eilidh McKemmie

Galois groups of random additive polynomials

Abstract: The Galois group of an additive polynomial over a finite field is contained in a finite general linear group. We will discuss three different probability distributions on these polynomials, and estimate the probability that a random additive polynomial has a "large" Galois group. Our computations use a trick that gives us characteristic polynomials of elements of the Galois group, so we may use our knowledge of the maximal subgroups of $\mathrm{GL}(n, q)$. This is joint work with Lior Bary-Soroker and Alexei Entin.

Tali Monderer

The reducible components of the curve $f(x) = g(y)$ for f indecomposable of large degree

Abstract: Families of rational functions f, g for which the curve $f(x) = g(y)$ is reducible give rise to interesting phenomena in number theory and complex analysis. It is known that no such indecomposable polynomials exist when $\deg(f) > 33$; for rational functions little is known. We describe the possible factorizations when f is taken to be rational, indecomposable of large degree, relying on an analysis of low genus coverings with Galois closure contained in that of f.

Alina Ostafe

Multiplicative and additive relations for values of rational functions and points on elliptic curves

Abstract: For given rational functions $f_1, \ldots, f_s$ defined over a number field, Bombieri, Masser and Zannier (1999) proved that the algebraic numbers α for which the values $f_1(\alpha), \ldots, f_s(\alpha)$ are multiplicatively dependent are of bounded height (unless this is false for an obvious reason). Motivated by this, we present various extensions and recent finiteness results on multiplicative relations of values of rational functions, both in zero and positive characteristics. In particular, one of our results shows that, given non-zero rational functions $f_1, \ldots, f_m, g_1, \ldots, g_n \in \mathbb{Q}(X)$ and an elliptic curve E defined over $\mathbb{Q}$, for any sufficiently large prime p, for all but finitely many $\alpha \in \overline{\mathbb{F}}_p$, at most one of the following two can happen: $f_1(\alpha), \ldots, f_m(\alpha)$ satisfy a short multiplicative relation or the points $(g_1(\alpha), \cdot), \ldots, (g_n(\alpha), \cdot) \in E_p$ satisfy a short linear relation on the reduction E_p of E modulo p.

Sebastian Petersen

Local to global principles for homomorphisms of abelian schemes

Abstract: Let A and B be abelian schemes over a smooth variety S/k. We establish criteria, in terms of restriction maps to subvarieties of S, for existence of various classes of homomorphisms from A to B, e.g., for existence of isogenies. Our main tools consist of Hilbertianity methods, Tate conjecture as proven by Tate, Zarhin and Faltings, and of the minuscule weights conjecture of Zarhin in the case, when the base field is finite.

Mark Shusterman

Arithmetic and geometry of tame extensions of global fields

Abstract: We consider analogs of several results and problems in Galois theory when restricted to tame extensions, highlighting phenomena that arise in this restriction.

Arvind Suresh

Realizing Galois representations in abelian varieties by specialization

Abstract: We present a strategy for constructing abelian varieties J/K which realize a given rational Galois representation $\rho : G_K \to \mathrm{GL}_n(\mathbb{Q})$, i.e. such that ρ is a subrep. of $J(\overline{K}) \otimes \mathbb{Q}$. When ρ is the trivial rep., J/K realizes ρ if and only if $J(K)$ is of rank at least n; such families are usually constructed by the specialization method pioneered by Neron. Our strategy consists in taking such a family and, provided there is enough symmetry, twisting the construction to obtain non-trivial Galois actions on the points. After twisting, we use a simple generalization of the classical Neron specialization theorem (from trivial reps. to non-trivial reps.) Applying this procedure to a construction of Mestre and Shioda, we prove: Given ρ, for any sufficiently large g, there exist infinitely many g-dimensional absolutely simple abelian varieties over K which realize ρ.

Olivier Wittenberg

Supersolvable descent for rational points

Abstract: The by now classical formalism of descent under a torus introduced by Colliot-Thélène and Sansuc in the 1980's admits an analogue in which the torus is replaced with a supersolvable finite group. I will explain this formalism and discuss applications to rational points of homogeneous spaces of linear groups over number fields and to the inverse Galois problem with prescribed norms. This is joint work with Yonatan Harpaz.

Umberto Zannier

Around effectivity problems in number theory

Abstract: I will briefly survey on the issue of effectivity in Number Theory, focusing especially on problems of integral points in diophantine analysis. In the last part of the talk I shall present a very brief account of some results recently obtained for some curves of genus 2 in work in progress with P. Corvaja.